EUROPEAN SOUTHERN OBSERVATORY

Organisation Européenne pour des Recherches Astronomiques dans l'Hémisphère Austral

Europäische Organisation für astronomische Forschung in der südlichen Hemisphäre

VLT PROGRAMME

VERY LARGE TELESCOPE

INSTRUMENTATION DIVISION

Effect of DM Dynamics on MACAO Performance

Doc. No. VLT-TRE-ESO-11640-2429
Issue: 1.0
Date: 31.12.2000

	Name	Date	Signature
Prepared:	E. Fedrigo	31.12.2000	
Approved:	J. Brynnel	31.12.2000	
Released:	N. Hubin	31.12.2000	

CHANGE RECORD

ISSUE	DATE	SECTION/PAGE AFFECTED	REASON/INITIATION DOCUMENTS/REMARKS
1	December 31, 2000	All	First Issue

1 Applicable Documents

AD 1	

2 Reference Documents

RD 1	Design Report MACAO-VLTI Deformable Mirror, CILAS, DPCO-00.1630, 13/12/2000
RD 2	MACAO-SINFONI Real Time Computer Software Specifications
RD 3	MACAO-VLTI Real Time Computer Software Specifications
RD 4	MACAO-VLTI System Overview
RD 5	AO Module System Overview

3 Acronyms

ESO	European Southern Observatory
CILAS	Compagnie Industrielle des LASers
AO	Adaptive Optics
AD	Applicable Document
RD	Referenced Document
MACAO	Multiple Application Concept for Adaptive Optics
SINFONI	SINgle Far Object Near-ir Investigation

VLT	Very Large Telescope
VLTI	Very Large Telescope Interferometer
DM	Deformable Mirror
HVA	High Voltage Amplifier

TABLE OF CONTENTS

4 Introduction

A central element in ESO's in-house developed Adaptive Optics System MACAO is the Deformable Mirror (DM). To the backside of this mirror several bimorph piezo elements are attached forming an array. By applying voltage to each of those piezo elements, the mirror profile can be adjusted thus reducing image distortion significantly. This mirror is characterised by a transfer function that exhibits a significant resonance.

A DM controller generates the high voltages required to drive the bimorph elements to the required deflection. This controlled is characterised by a low-pass transfer function.

4.1 Purpose

Simulations to forecast MACAO-VLTI performance have always been carried out without simulating the dynamic of the deformable mirror.
This approximation can be considered valid as long as the DM dynamics is *large* in respect to the control loop frequency, where large means that the bandwidth at –3dB is significantly bigger than the loop frequency.
In case of MACAO-VLTI the first resonant frequency is located at 590 Hz (estimate from CILAS, the DM manufacturer) and, consequently, the bandwidth of the DM is not significantly bigger than the loop frequency that runs at 350 Hz for bright stars.
The purpose of this memo is to show how the simulation has been modified to add the mirror dynamics and to present the results.

4.2 Scope

This document analyses the effect of the DM resonant modes on the performance of the system under different points of view:

- Speed of the system: analysis of the step/impulse response
- Image quality: final Strehl ratio.
- Piston contribution: the influence of the resonant modes on the amount of added piston.

We present analytical models of the DM dynamics based on two sets of data:

- Desired transfer function, according to the specifications.
- Expected transfer function, according to the first report received from the DM manufacturer (CILAS)

4.3 System Architecture

In the following we will refer to the MACAO system that is currently under design with two flavours: MACAO for VLTI and MACAO for SINFONI. The mirror for the two systems is different and also the problems that the resonant mirror modes can rise, but the architecture is identical.

MACAO is composed by several hardware and software subsystems ranging from coordination and interface to the real time control of the active optical elements. In this work we are mainly interested in the real-time part of MACAO, the so-called *corrective optics* and its control.

The optical path is modified by the active optical element called Deformable Mirror that, for MACAO, consists of a curvature bimorph mirror with 60 actuators mounted in a tip/tilt stage. The curvature mirror is based on bimorph piezoelectric technology. The DM responds to voltages in the range of –400 to +400 Volts with a mechanical deformation to compensate for atmospheric aberrations.
The DM is mounted in the tip/tilt stage such that the centre of rotation and centre of gravity is on the surface of the DM to avoid introduction of piston errors during operation. The tip/tilt stage is used to offload high stroke tip and tilt.

In order to be controlled the DM is connected to a High Voltage Amplifier that can provide the voltages in the required range from a low-voltage analogical signal. The HVA is a complete subsystem composed by a bank of amplifier with a supervisory computer which is based on a ESO standard hardware.

Data about the configuration of the incoming wavefront are collected from a 60-elements curvature sensor: the light is divided into 60 subapertures and then counted photon by photon with a bank of APDs. From two independent sets of measures in particular optical conditions is possible to recover the instantaneous wavefront curvature (see Roddier for more details).

These data are collected by the Real Time Computer which computes in real time (i.e.: in 0.5ms) the configuration of the DM that minimise the spatial variance of the residual wavefront that results from the reflection against the DM.

The discussion in the present work is more tied to MACAO-VLTI in term of mirror transfer function and relevance of the resonance mirror mode problems like the piston.

5 Model of the Mirror Dynamics (Desired)

Each channel of the DM has a transfer function from voltage applied to the position reached which has a resonant frequency of 590 Hz (3707 rad/s) and an overshoot of 20dB (desired transfer function).
We model this transfer function with a second order system, which gives[1]:

$$H(s) = \frac{12500000}{s^2 + 360s + 1.377 \cdot 10^7}$$

The Bode plot is shown on Figure 5-1.

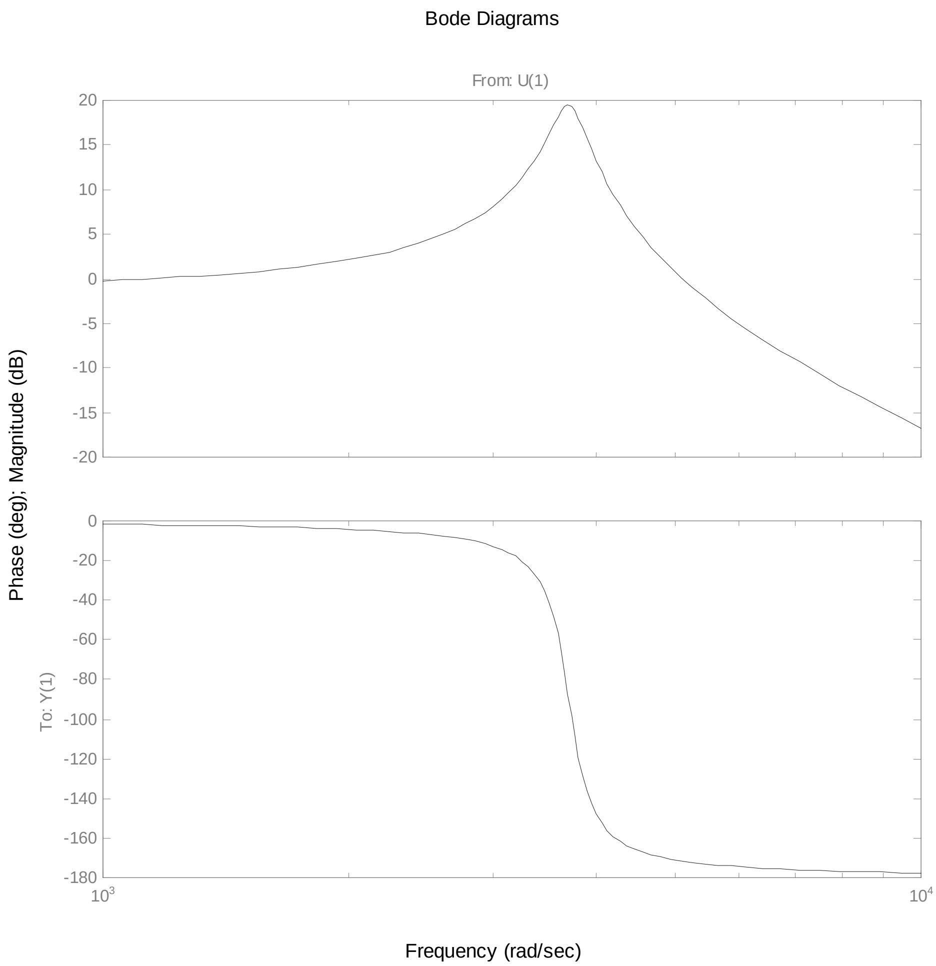


Figure 5-1: Bode plot for one DM channel

[1] File BIM60sim_s

Poles (rad/s)	Zeros (rad/s)	Gain
-180 ± i 3707	0	12500000

We have to transpose this transfer function into the digital domain, since our simulation is unable to handle the continuous-time domain.
The pace of the simulation is given by the membrane. For each membrane cycle the simulation computes 16 points. Given that the membrane mirror for VLTI vibrates at 2.1 kHz, the simulation is running at 33.6 kHz, which corresponds to an oversampling of 96 in respect to the control loop that runs at 350 Hz.

For every point a step of the evolution of the DM and the resulting residual wavefront have to be computed. Moreover, an optimisation that cut half of the computation cannot be used, due to the fact that the mirror is evolving during this time. This will slow down the simulation speed from 2 to 3 times.

Converting to the digital domain we have[2]:

$$H(z) = \frac{0.0055107(1+0.9964\mathrm{z}^{-1})}{1-1.977\mathrm{z}^{-1}+0.989\mathrm{z}^{-2}}$$

whose Bode plot is given on the next page, Figure 5-3 and the pole-zero map and table hereby (Figure 5-2).

Poles (rad/s)	Zeros (rad/s)	Gain
0. 988609 ± i 0. 109517	-0.99643349643014	0.00551074
	0	

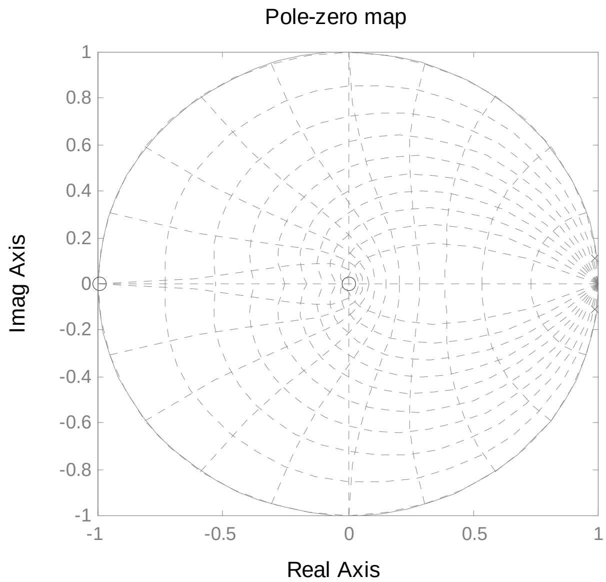


Figure 5-2: Pole-Zero map for the digital model of a single DM channel.

[2] File BIM60sim_z

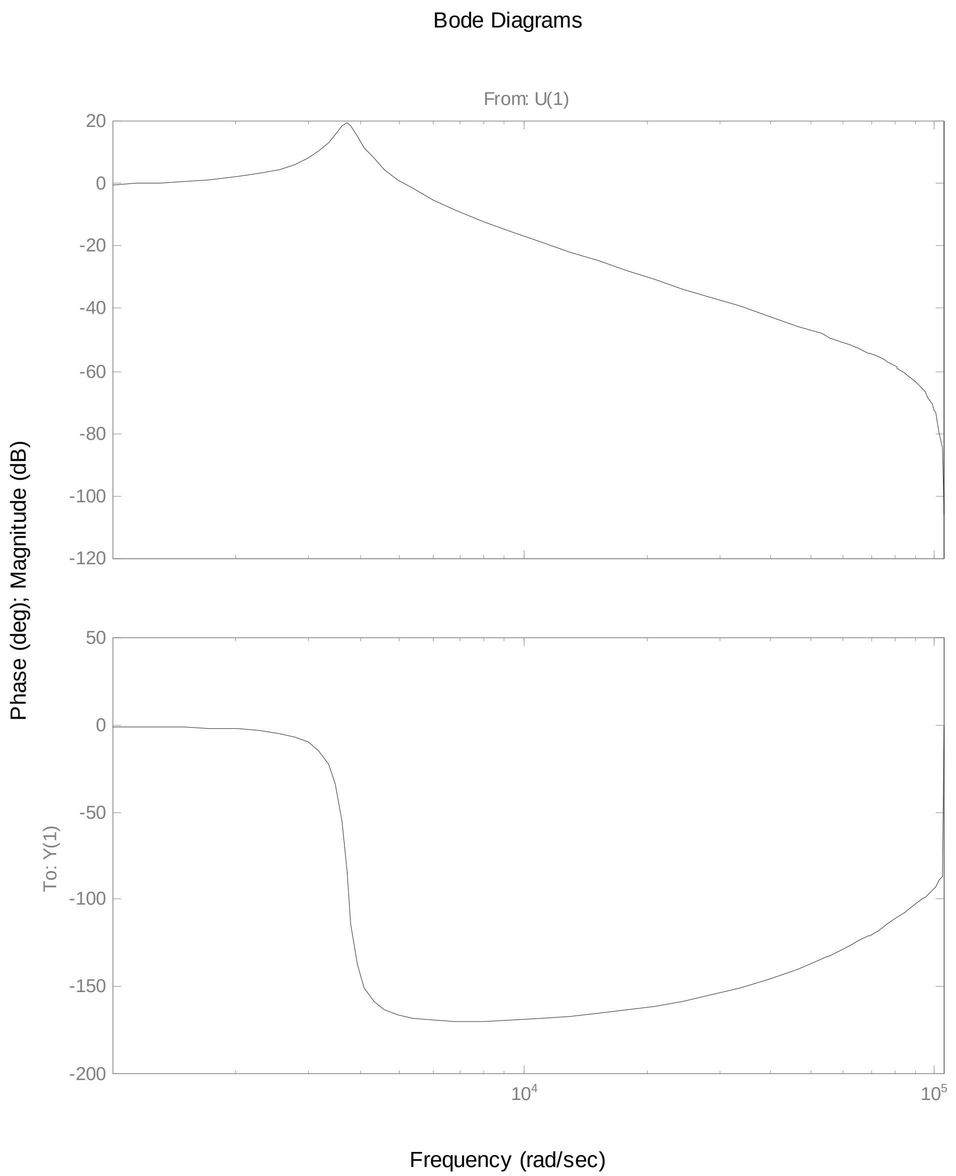


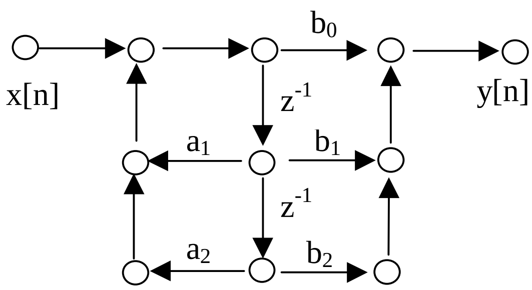
Figure 5-3: Bode plot of the digital version of the DM transfer function (single channel)

5.1 Simulation Model

The simulation produces one sample each 2.86 ms. In this period the mirror will feed the digital filter that emulates the mirror dynamics 16 times (33.6KHz).
Thus the HVA is modelled without any dynamics and acts as an ideal holder.
The filter is implemented with a direct realisation, since no round-off noise and quantization effects are expected using double precision arithmetic.
The direct form II structure is then:

$$y[n] = \sum_{k=0}^{2} b_k x[n-k] + \sum_{k=1}^{2} a_k y[n-k]$$

Num		Den	
b_0	0.00551074714547	a_0	1
b_1	-1.97721902322505	a_1	-1.97721902322505
b_2	0.98934290780010	a_2	0.98934290780010



The matlab code for the filter is the following:

```
a  = [a1,a2];
b  = [b0,b1,b2];
s  = [0,0];
ys = [0,0,0];

% status update: input
s(2:length(b),:)=s(1:length(b)-1,:);
s(1,:) = x;

% one-step IIR filter for the DM dynamics
y = b*s- a*ys;

% status update: output
ys(2:length(a),:)=ys(1:length(a)-1,:);
ys(1,:) = y;
```

5.2 Analysis

Given this desired model we can reach some conclusion on the desired DM behaviour.

We first analyse the dynamic model we computed. The following figure shows the impulse response of such a system:

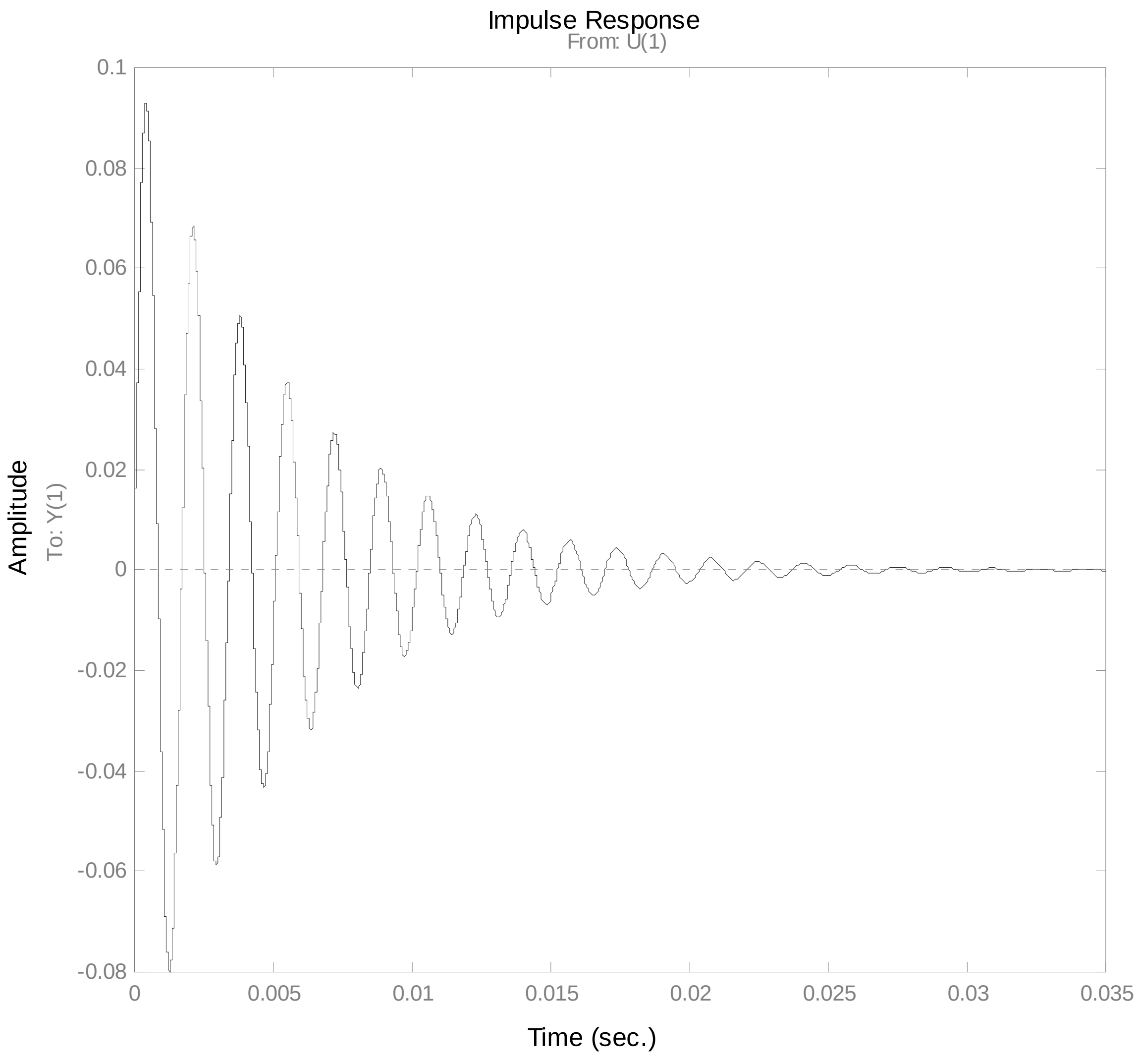


Figure 5-4: DM Impulse Response – desired case

The system is quite slow to stabilise. In fact, it takes more than 20 ms.
We can see the effect of this slowness applying to this filter one signal coming from the control loop. Let's suppose the control loop is running as fast as possible, i.e.: 350 Hz. This means that one sample is sent to the mirror each 6 membrane cycles which, in turn, is made of 16 samples.

This result in a holder that map a signal at 350 Hz to one at 33.6KHz holding the last value for 6*16 = 96 samples.
We apply this signal to the filter that models the DM and we show the results in Figure 5-5.

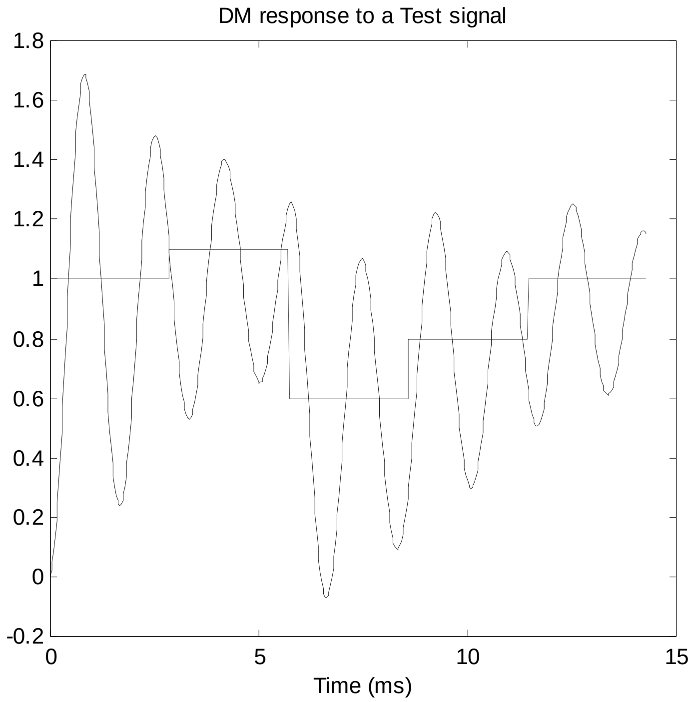


Figure 5-5: DM Response to a Signal

The system is almost always oscillating and the oscillation is proportional to the given step. This means that during the bootstrap phase the oscillation will be larger than in close-loop regime.

To correct this behaviour we need to insert a controller or, better, a signal shaper in order to reduce the effect of the resonant frequency at the cost of a reduction in the system speed.
We would like a transfer function and impulse response similar to the one shown in Figure 5-6 which doesn't have the overshoot and also a reasonable set time. The step response is quite smooth and this can help to control the amount of piston introduced.
This filter has the following transfer function[3]:

$$H_1(z) = \frac{0.018665(1+0.8877\mathrm{z}^{-1})}{1-1.663\mathrm{z}^{-1}+0.6997\mathrm{z}^{-2}}$$

The compensator we are looking for is, therefore[4]:

[3] File BIM60sim_s2 and BIM60sim_z2
[4] File BIM60shaping_z, concatenation of the two in BIM60desired_z

$$H_c = H_1 / H = 3.38704 \frac{1 + 0.8877z^{-1}}{1 - 1.663z^{-1} + 0.6997z^{-2}} \cdot \frac{1 - 1.977z^{-1} + 0.989z^{-2}}{1 + 0.9964z^{-1}} =$$

$$= 3.38707 \frac{1 - 1.0893z^{-1} - 0.7659z^{-2} + 0.8779z^{-3}}{1 - 0.666z^{-1} - 0.9573z^{-2} + 0.6971z^{-3}}$$

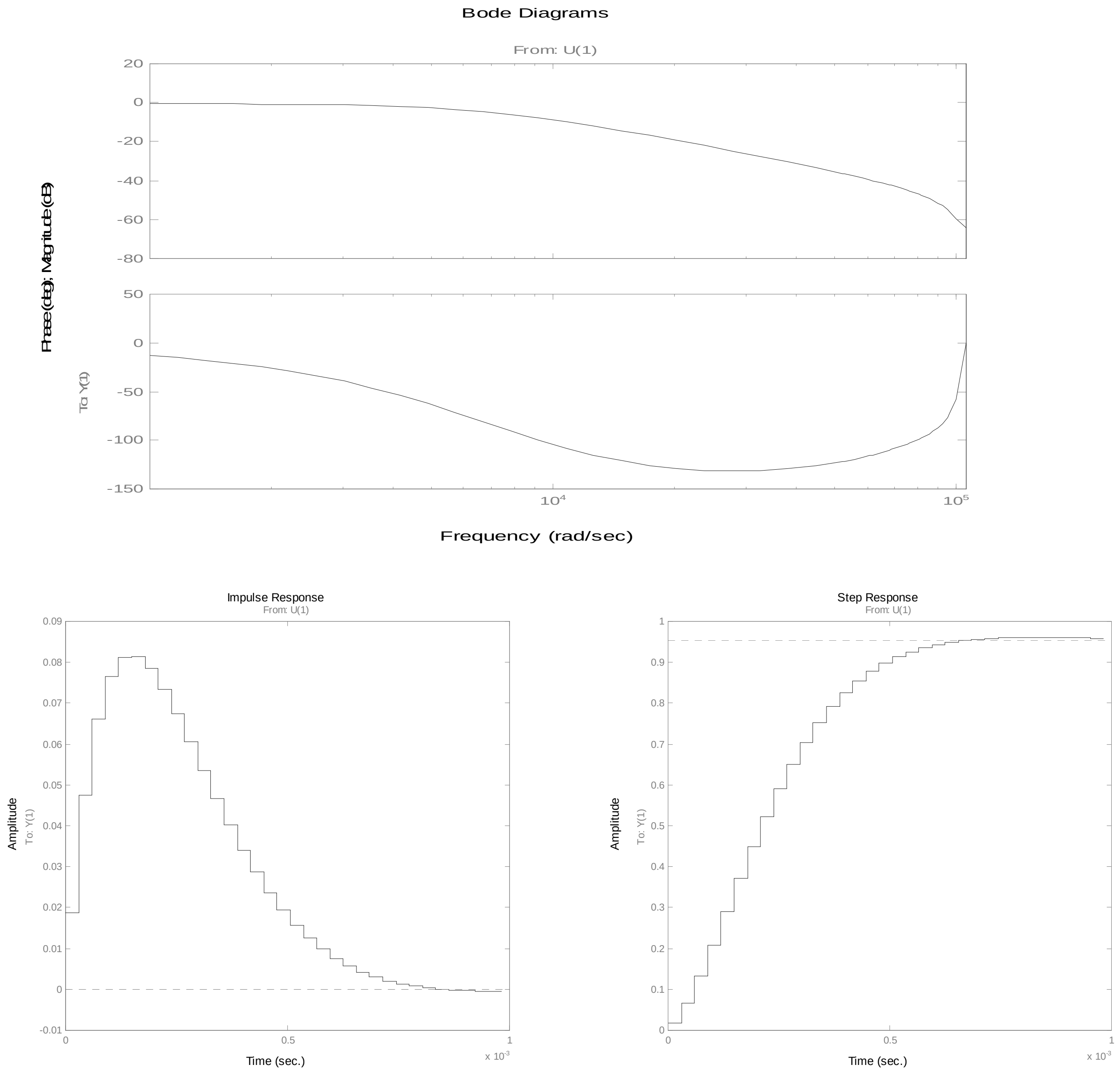


Figure 5-6: Desired DM response

This compensator is in fact a notch tuned to the resonant frequency. It is described as:

Num		Den	
b_0	3.38696763673360	a_0	1
b_1	-3.69013126223742	a_1	-0.66632345355181
b_2	-2.59392442683669	a_2	-0.95715418400891
b_3	2.97460348099965	a_3	0.69717715277285

Poles (rad/s)	Zeros (rad/s)	Gain
0.8313784 ± i 0.0920997	-0.88771015	3.386967
-0.9964334	0.9886095 ± i 0.10951776	

so, it is stable.

The result to same signal is then:

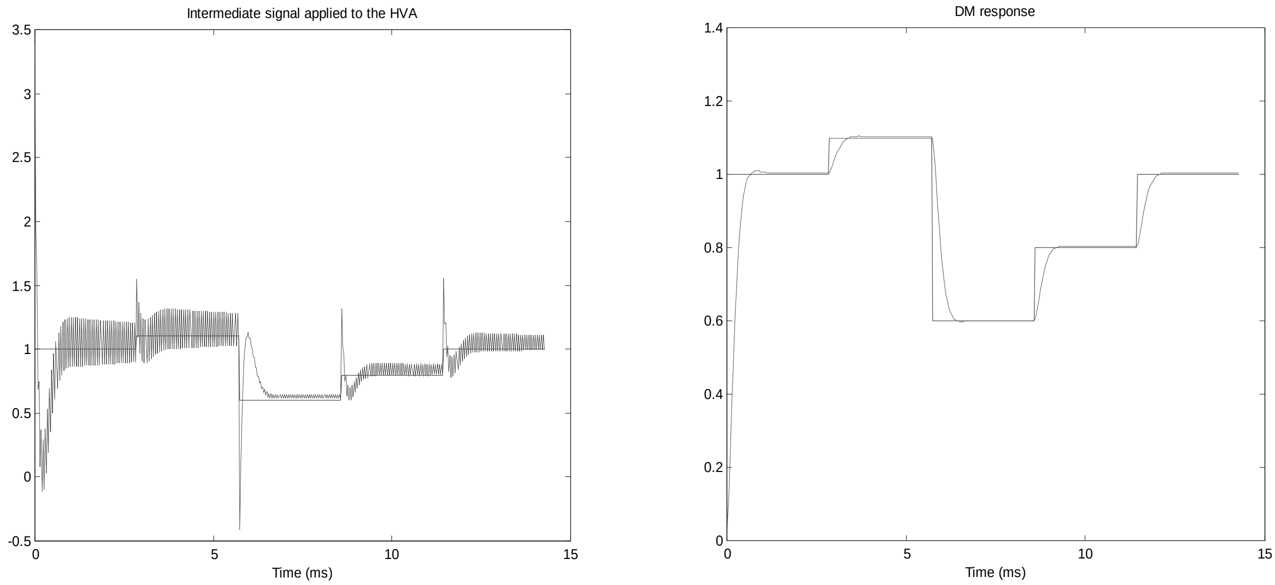


Figure 5-7: Response of the DM with the compensator. On the left, the intermediate signal sent to the DM, on the right the resulting DM response.

In this example, the HVA is assumed to have infinite bandwidth. The finite bandwidth of the HVA will cut off part of the information contained in the intermediate signal. This effect will be analysed later.
Of course the specs of the desired response can be relaxed resulting in a less demanding shaping filter that can accept a low-pass filter before the DM (the HVA) and also can run at lower sampling rate (computing power issue).

6 Model of the Mirror Dynamics (Expected)

RD1 shows that the DM we are going to receive is slightly different than expected.
That report shows the transfer function of a DM made with the same technology but different size and number of electrodes (BIM13). It is characterised by 2 significant resonant frequency, the one as in the desired model plus an additional resonant mode at a lower frequency.
We have been given the following table which describes the DM for VLTI (150mm, 100mm pupil):

Model	Diameter (mm)	Thickness (mm)	f1 (Hz)	Gain (dB) at resonance f1	f2 (Hz)	Gain (dB) at resonance f2
BIM60	150	2.6 (2 PZT and 0.6 glass)	~250	~8	810	~25

Trying to fit these data to obtain a transfer function similar to the one provided for BIM13 we obtain the following sixth-order model[5]:

$$H(s) = \frac{b_0 + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4 + b_5 s^5 + b_6 s^6}{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5 + s^6}$$

Num		Den	
b_0	2.9506e+020	a_0	2.9495e+020
b_1	3.5415e+016	a_1	6.1826e+016
b_2	1.3074e+014	a_2	1.9753e+014
b_3	8.3159e+009	a_3	2.0720e+010
b_4	1.2993e+007	a_4	3.3165e+007
b_5	0	a_5	801.2434
b_6	0	a_6	1

Bode plot is given in Figure 6-2, pole-zero map in Figure 6-1 and table hereby.

Poles (rad/s)	Zeros (rad/s)	Gain
-156.325650 ± i 1570.79632	-81.142306 ± i 1870.245688	1.2993e+007
-181.087200 ± i 2129.84998	-238.90942 ± i 2534.443624	
-63.2088441 ± i 5089.38009		

[5] Matlab file BIM60tf_s

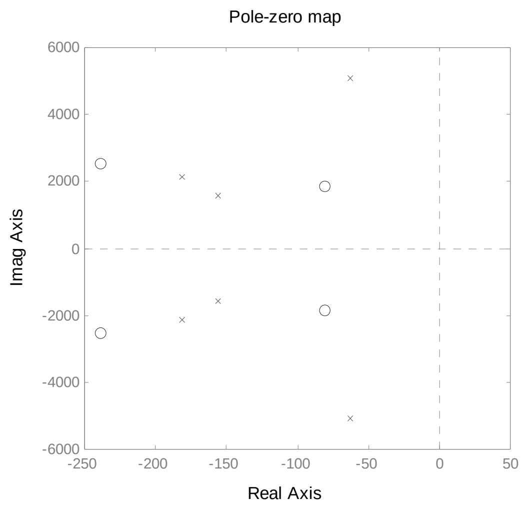


Figure 6-1: Pole-Zero map for one DM channel, continuous time domain, "real" mirror.

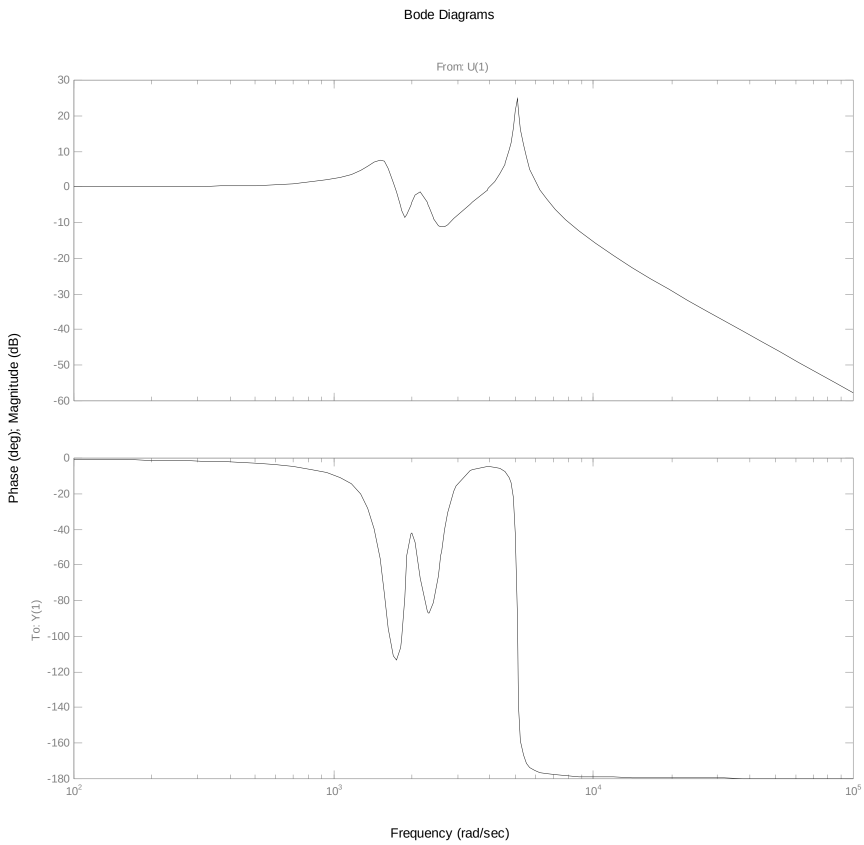


Figure 6-2: Bode plot for one DM channel, continuous time domain, "real" mirror

As we did in the previous section, we have to transpose this transfer function into the digital domain in order to add the DM dynamics to our simulation. Same considerations apply so we design our transfer function with a sampling rate of 33.6 kHz.

Converting to the digital domain we have[6]:

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

whose Bode plot is given on the next page, Figure 6-4, and the pole-zero map and table hereby (Figure 6-3).

Poles (rad/s)	Zeros (rad/s)	Gain
0.994270 ± i 0.046515	0.990091± i 0.074824	0.00573528767420
0.992627 ± i 0.063005	0.996043± i 0.0554991	
0.986692 ± i 0.150607	-0.998402	

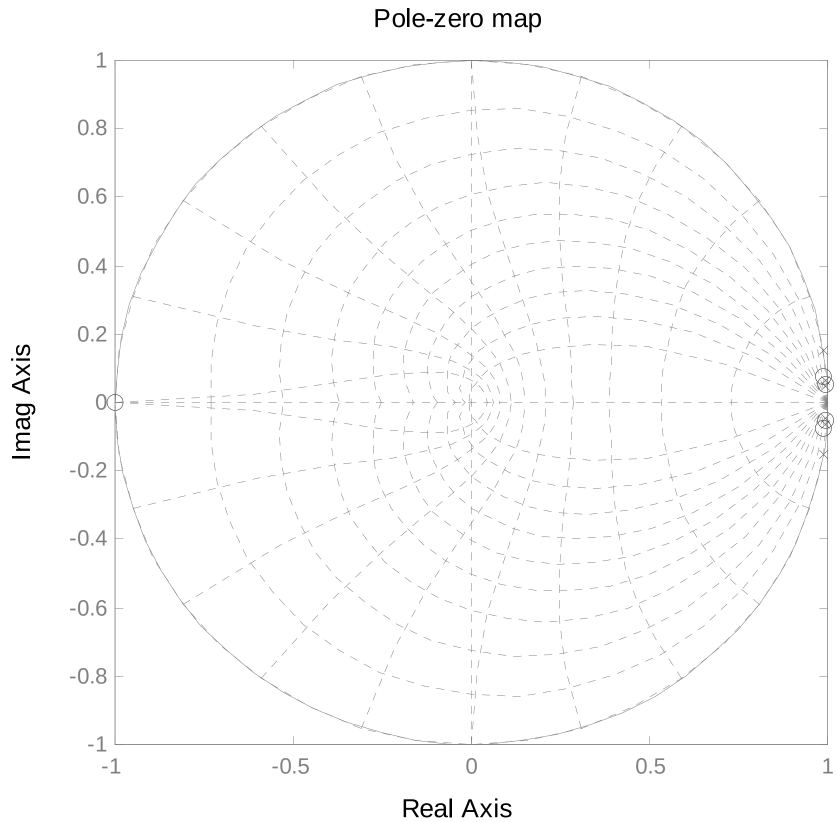


Figure 6-3: Pole-Zero map for the digital model of the real DM (single channel)

[6] Matlab file BIM60tf_z

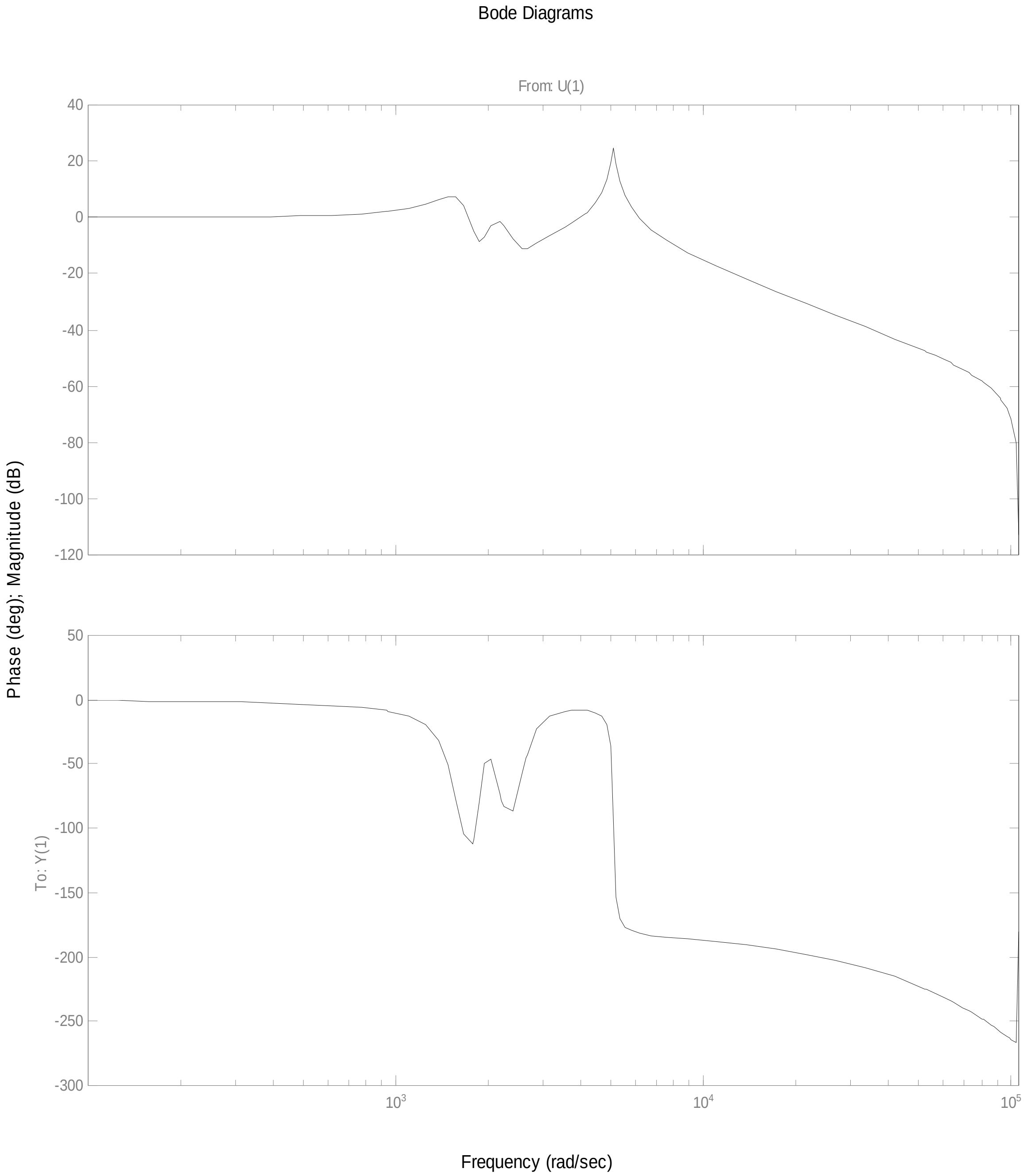


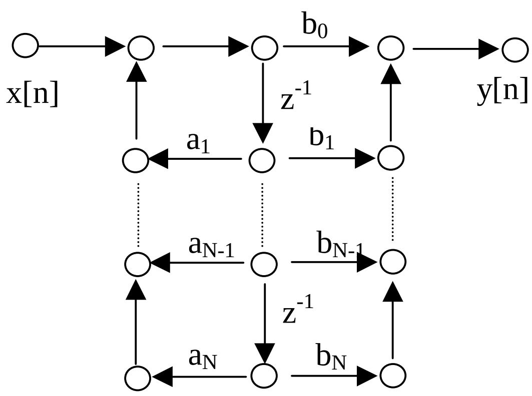
Figure 6-4: Bode plot of the digital version of the real DM transfer function (single channel)

6.1 Simulation Model

With the same considerations given in section 5.1, the direct form II structure is then:

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] + \sum_{k=1}^{N} a_k y[n-k]$$

Num	Den		
b_0	0	a_0	1
b_1	0.00573528767420	a_1	-5.94718126622914
b_2	-0.01705598899828	a_2	14.76585301793066
b_3	0.01124022564947	a_3	-19.59075184463729
b_4	0.01136556443661	a_4	14.64910708038053
b_5	-0.01690296725973	a_5	-5.85346233773063
b_6	0.00561808063636	a_6	0.97643555234386



The matlab code for the filter is the following:

```
a  = [a1,a2,…,aN];
b  = [b0,b1,…,bM];
s  = [0,0, …, 0];
ys = [0,0, …, 0];

% status update: input
s(2:length(b),:)=s(1:length(b)-1,:);
s(1,:) = x;

% one-step IIR filter for the DM dynamics
y = b*s- a*ys;

% status update: output
ys(2:length(a),:)=ys(1:length(a)-1,:);
ys(1,:) = y;
```

6.2 Analysis

Given this real model we can reach some conclusion on the real DM behaviour.

We first analyse the dynamic model we computed. The following figure shows the impulse response of such a system:

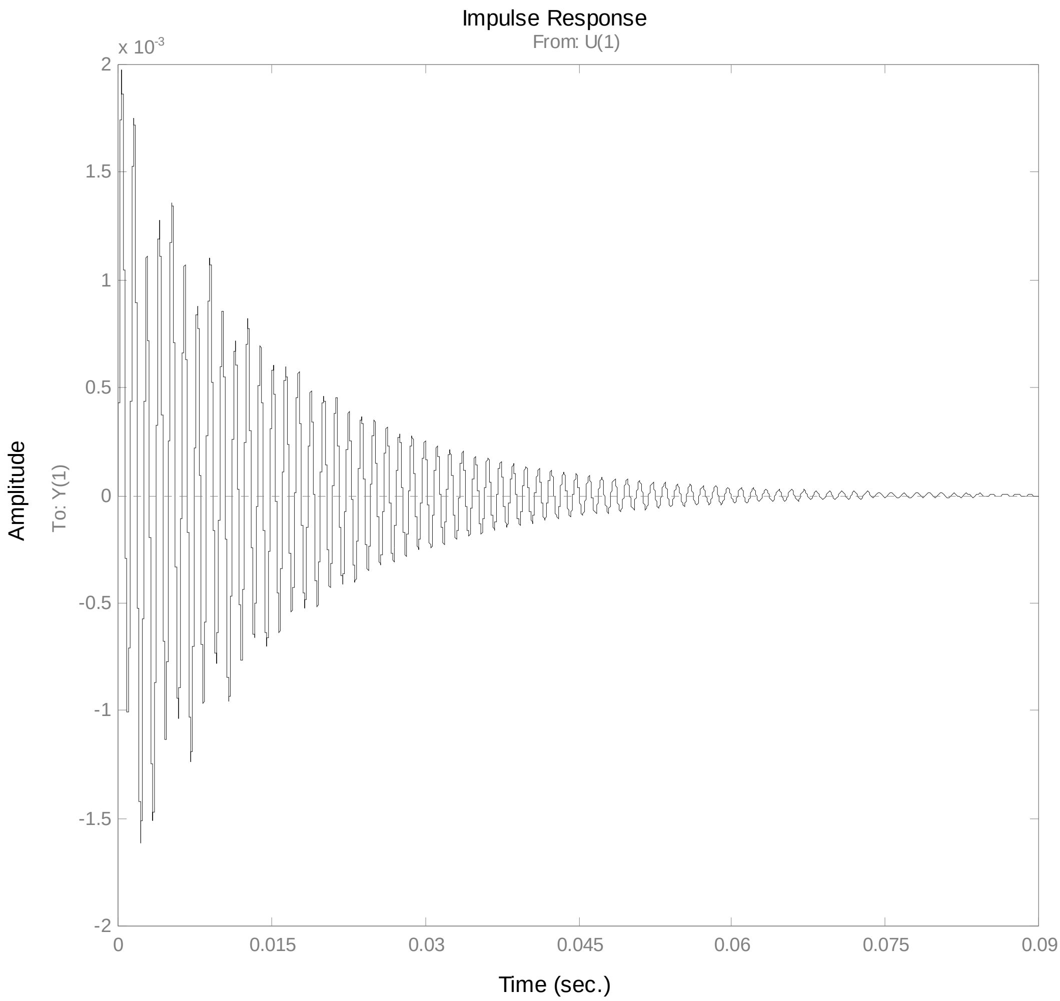


Figure 6-5: DM Impulse Response – real case

The system is slower to stabilise than the desired case. In fact, it takes more than 50 ms compared to 20 ms. Applying the same signal as done in the previous section we obtain the response shown in Figure 6-6, compared with the response of the desired case (dotted line).

The system is clearly slower but the behaviour is not dramatically different from the previous case.

We then apply the same considerations given before and we design the same shaping filter, centered at a different frequency.

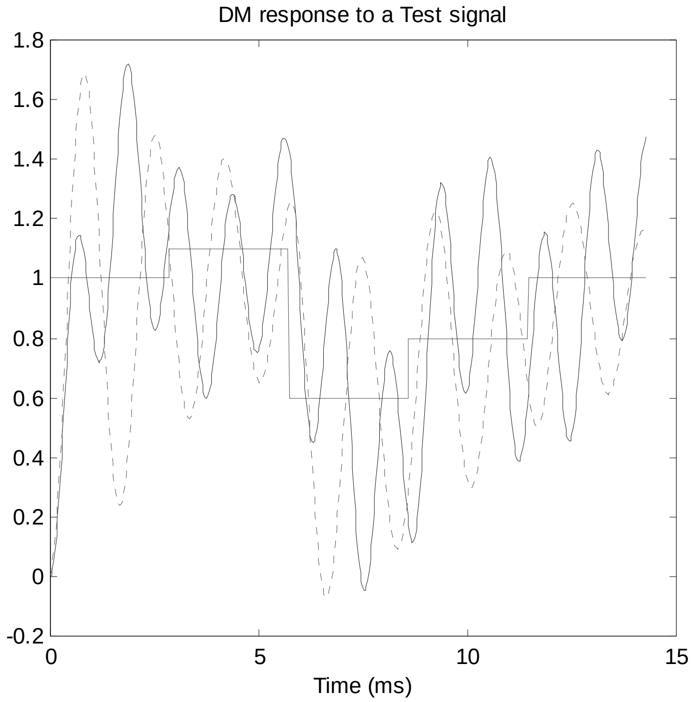


Figure 6-6: DM Response to a Signal

The parameters of the filter are[7]:

Num		Den	
b_0	2.11685477295850	a_0	1
b_1	-2.28366853349832	a_1	-0.65731998170217
b_2	-1.60108414278520	a_2	-0.94818282307375
b_3	1.85912717562478	a_3	0.69717715277285

Poles (rad/s)	Zeros (rad/s)	Gain
$0.826876 \pm i\ 0.126282$	-0.88771015	2.1168547
-0.996433	$0.983256 \pm i\ 0.150165$	

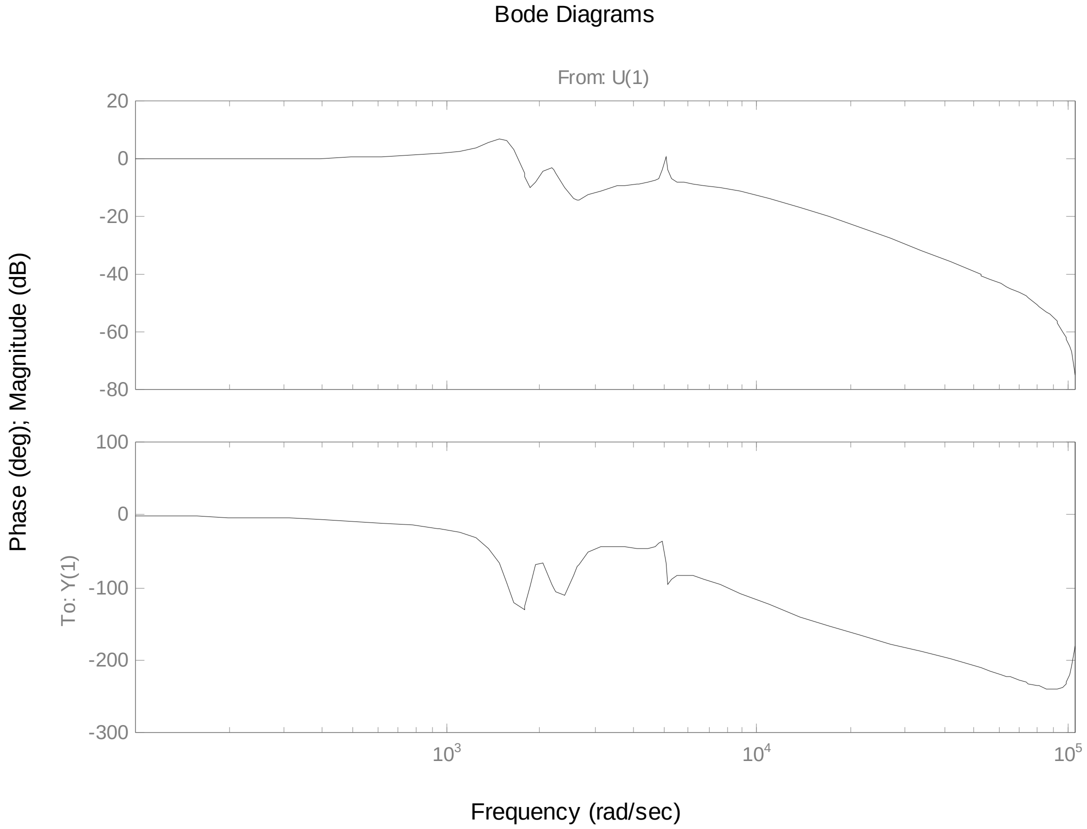
so, it is stable.

The cascade of the two filters, the resulting system, will have the transfer function, step and impulse response shown in Figure 6-7.

Applying the same signal as we did in the previous sections, without taking into account the HVA low-pass behaviour, we obtain what shown in Figure 6-8. In dotted line there is the response of the desired case for a comparison. It is not as good as in the previous case because we are limited by the number of coefficients of

[7] Matlab file BIM60shaping2_z

the shaping filter (still 3rd order). Of course, using a 6th or 7th order IIR filter one can match exactly the DM transfer function and obtain the same results as before. But, of course, the final result is dramatically dependent on the accuracy of the transfer function measurement and the tuning of a 7th order filter if naturally more complicated than the tuning of a 3rd order filter.



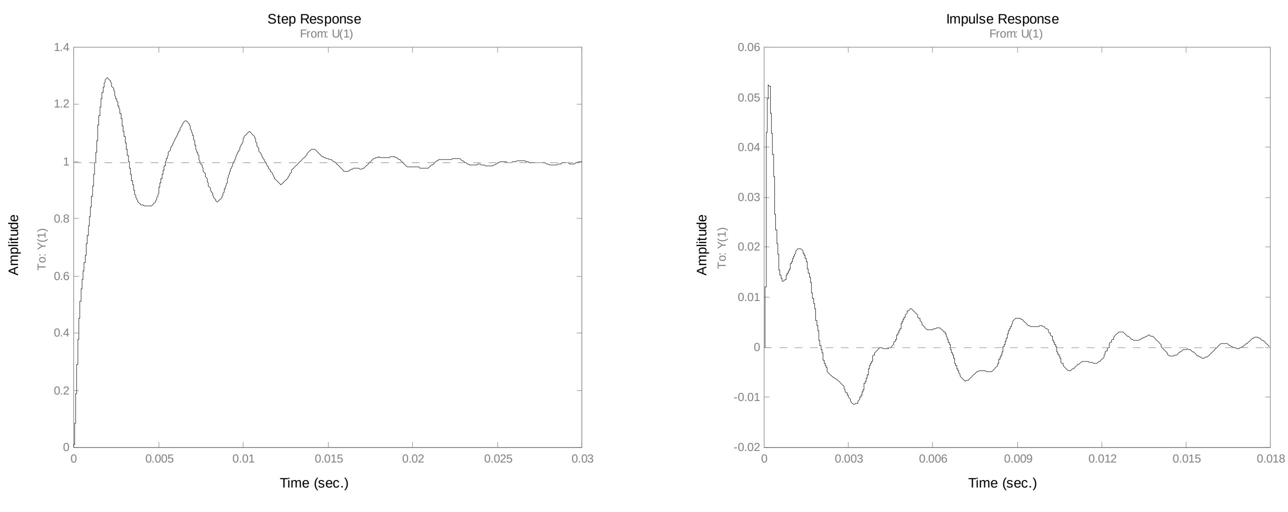


Figure 6-7: Compensated DM response

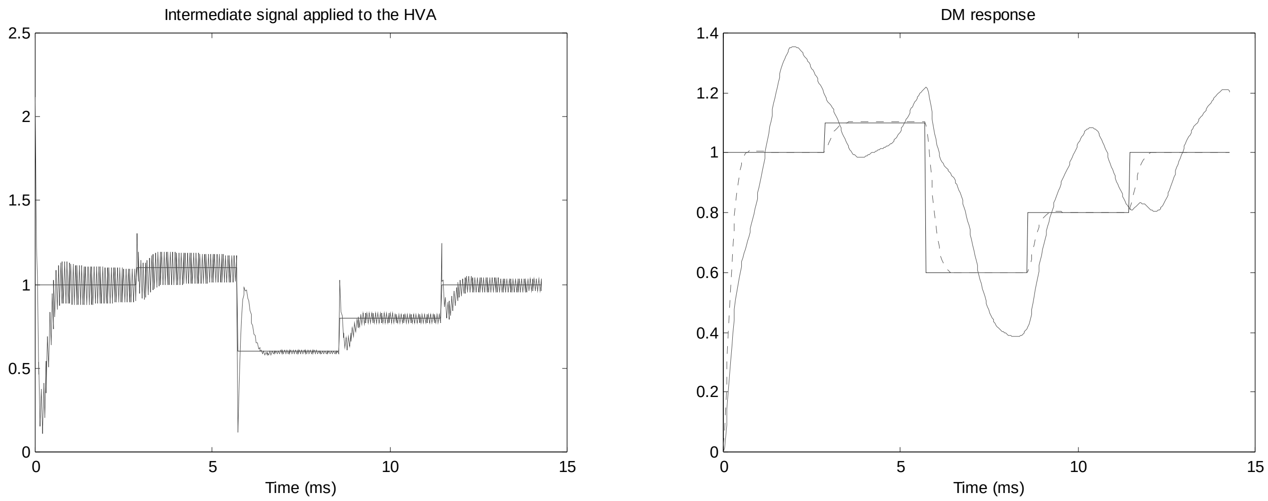


Figure 6-8: Response of the real DM with the compensator. On the left the intermediate signal sent to the DM, on the right the resulting DM response.

7 Shaping Filter

The selected shaping filter is quite simple to tune because it needs only 2 parameters: the resonant frequency and the amount of overshoot to compensate for.
In this section we analyse some issues about this filter like the presence of a low-pass filter in the chain, the need to lower the operating frequency and the effect of a bad tuning.

7.1 HVA Bandwidth

The HVA is characterised by a low-pass filter behaviour. It can be modelled as a first-order system with a cut-off frequency of 4 kHz. The model is then[8]:

$$H(s) = \frac{25130}{\mathrm{s} + 2.513\mathrm{e}004}$$

which in digital domain (33.6kHz) turns to[9]:

$$H(z) = \frac{0.52663}{\mathrm{z} - 0.4733}$$

The effect of this filter on the performance of the shaping filter is shown in the next figures. It is clear that the low-pass filter behaviour introduced by the HVA doesn't affect the performance of the shaping filter.

[8] Matlab file HVA_s
[9] Matlab file HVA_z

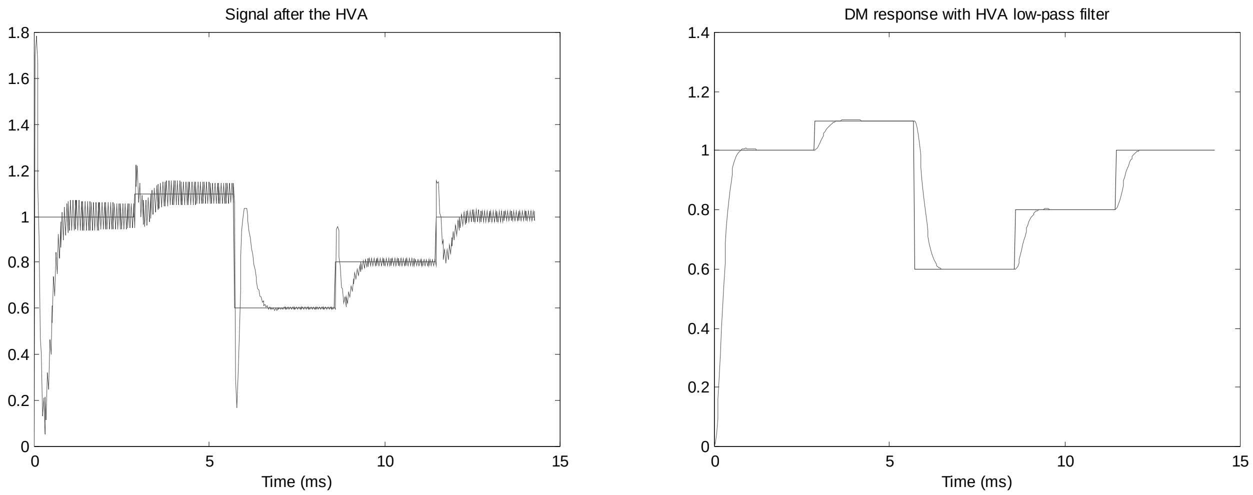


Figure 7-1: Response of the desired DM, with HVA low-pass filter

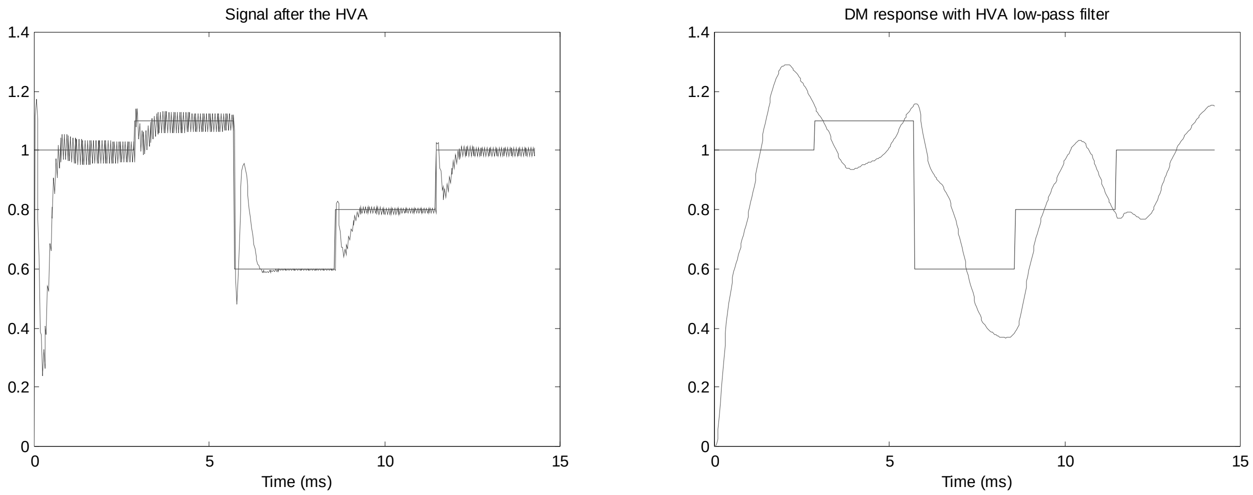


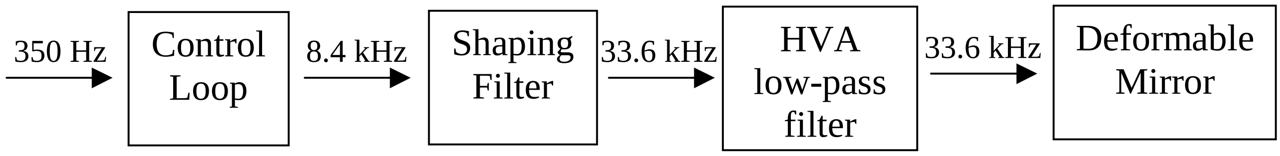
Figure 7-2: Response of the real DM, with HVA low-pass filter

7.2 Low-Frequency Shaping Filter

Running the filter at 33.6 kHz may be too demanding for a computer not entirely dedicated to the filter implementation, so we need to investigate other designs of the same filter which run at a lower frequency. Since the highest resonance frequency to control is about 800 Hz, we choose to reduce the sampling rate around 8 kHz. In fact we undersample the previous case by a factor of 4, giving a sampling rate of 8.4 kHz.

To implement it we simply oversample the mirror commands with a factor of 24 instead of 96, we apply the filter and then we oversample again with a factor of 4 to get back to 33.6 kHz.

With a block diagram:



where every change of frequency is realized with an sample-and-hold interpolator.
To design the shaping filter at 8 kHz we repeat the conversion from continuous to discrete time done in the previous paragraphs, but at a different frequency.
We determine first the digital equivalent (@ 8.4kHz) for the desired DM and the compensated transfer function, which are[10]:

$$H_{DM}(z) = \frac{0.085918\, z^{-1}(1+0.9857z^{-1})}{1-1.77z^{-1}+0.958z^{-2}} \qquad H_{comp}(z) = \frac{0.22088\, z^{-1}(1+0.6187\, z^{-1})}{1-0.8853\, z^{-1}+0.2397\, z^{-2}}$$

The shaping filter at 8.4 kHz is then[11]:

$$H(z) = 2.57085\frac{1-1.15137z^{-1}-0.13701z^{-2}+0.59271z^{-3}}{1-0.10044z^{-1}-0.63298z^{-2}+0.23622z^{-3}}$$

In Figure 7-3 the response is shown. A small ripple is present but the response is acceptable.

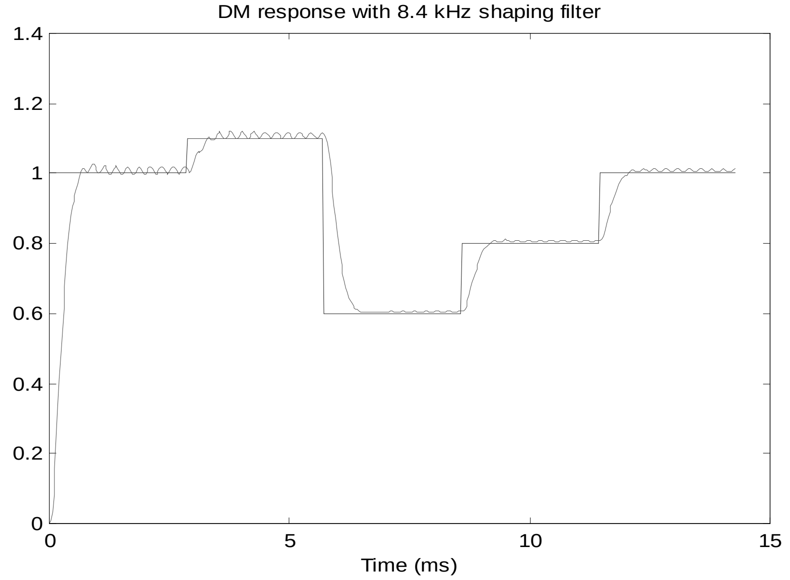


Figure 7-3: DM response (desired case) with a shaping filter runnning at 8.4 kHz

[10] Matlab files BIM60sim_z8 and BIM60sim2_z8
[11] Matlab file BIM60shaping_z8

Repeating for the real DM we get to this result for the shaping filter[12]:

$$H(z) = 2.57085 \frac{1 - 1.15137z^{-1} - 0.13701z^{-2} + 0.59271z^{-3}}{1 - 0.10044z^{-1} - 0.63298z^{-2} + 0.23622z^{-3}}$$

In Figure 7-4 the response is shown. A small ripple is present but the response is similar to the previous case.

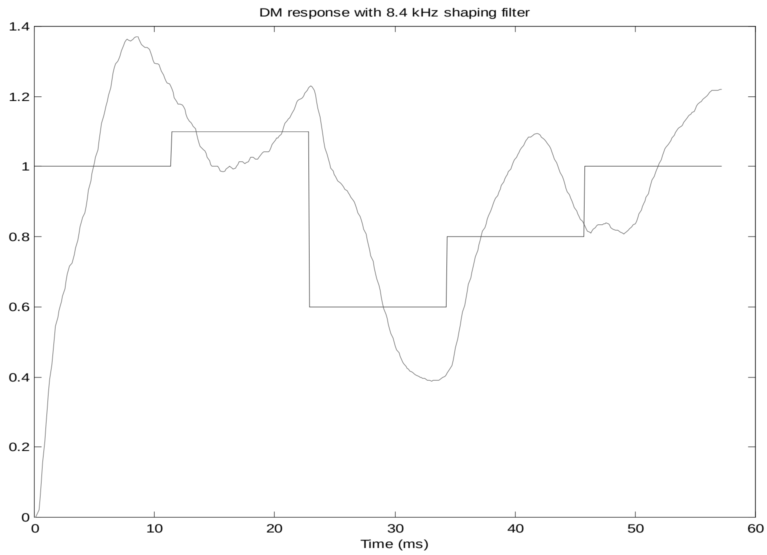


Figure 7-4: DM response (real case) with a shaping filter runnning at 8.4 kHz

7.3 Tuning the Filter

We now investigate the effect of a wrong tuning of the filter, only for the desired case where the matching of the filter to the system to compensate for is tighter.
This filter has 3 zeros and 3 poles. Four of them create the inverted resonance and have to be moved together to change the resonance frequency and overshoot. The other couple zero-pole *works* at high frequency. Changing the angle in which these poles and zeros are aligned we select the resonant frequency, varying the distance between the poles and the zeros we can change the overshoot. The position of the 2 zeros in this double zero-pole couple gives the width of the resonance but this can be pre-set and left there.

In Figure 7-5 and Figure 7-6 we can see this effect. Putting on the chain a detuned shaping filter can have very bad results adding a lot more noise than what we are correcting for. Of course, the problem is worse when the shaping filter is designed to perfectly match a particular transfer function, as in the case of Figure 7-5.

[12] Matlab file BIM60shaping2_z8

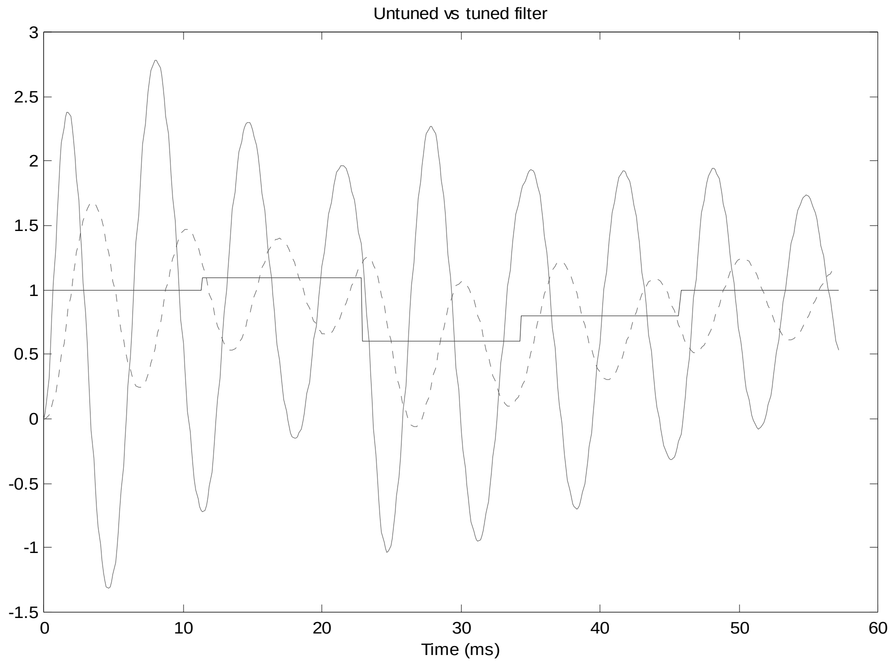


Figure 7-5: Effect of bad tuning of the shaping filter on the desired DM

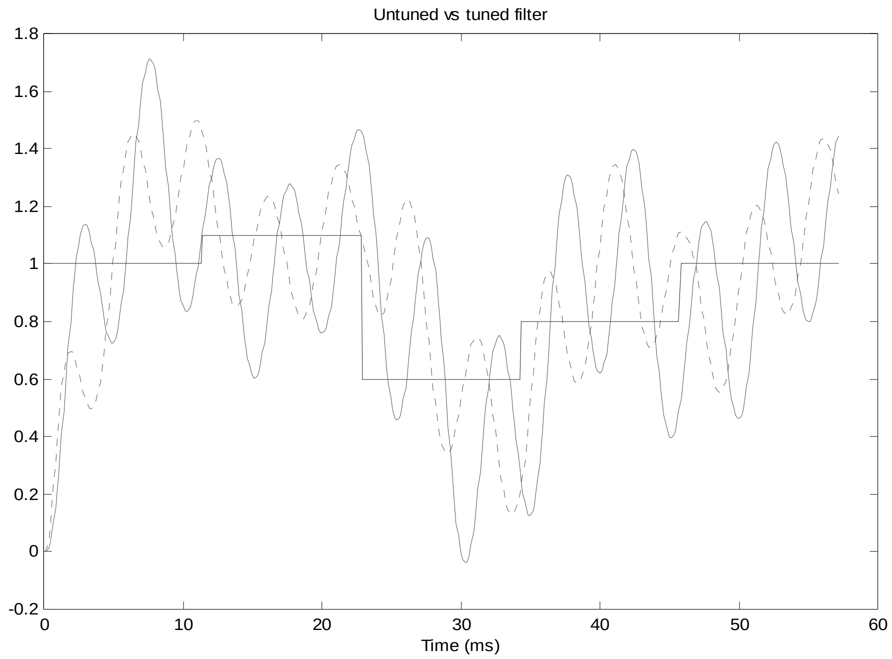


Figure 7-6: Effect of bad tuning of the shaping filter on the real DM

8 Simulations

The simulations are made under an important assumption: the transfer functions of all the electrodes are all equal, which is, of course, not true for the effect of the 3-point support. Being all equal, they oscillate in phase.
Each transfer function will be unique and it is required to properly compute the dynamic behaviour of the DM. Moreover, these transfer functions need to be known in order to design the shaping filter. The higher the order of the shaping filter, the better the compensation but also the better the required precision to measure the transfer functions of each electrode. Ageing problems and temperature drifts can change these transfer functions making the shaping filter no more effective (see section 7.3).

For simplicity the dynamic model has been applied to the voltages and not to the mirror position. Thus the simulation of the mirror is made of two pieces: a driving stage which outputs voltages connected to a mirror which respond instantaneously. The dynamic is simulated in the driving stage that outputs voltages with the proper dynamics. The figures in this section will show the dynamic of this driving stage, showing voltages instead of meters.

8.1 Mirror Dynamics

We have seen what is the effect of the mirror dynamic on the step response, which the basic command sent to the mirror on each channel at the loop frequency.
The following figures show the evolution of the voltage for the central electrode for the three cases considered:

- Ideal case (no dynamics), blue line.
- Simplified dynamics (desired case), green line.
- Complete dynamics (real case), red line.

for stars with m_v=10 and m_v=15.5 and seeing (@500nm) equal to 0.65” and 1” and without any shaping filter.

In the first two cases (bright star) the effect of the dynamics is very small and the evolution of the voltages with dynamics is very close to the ideal case without it.
Bigger effects are present in the case of faint star, probably because each single step commanded by the control loop is bigger because the integration time is longer.

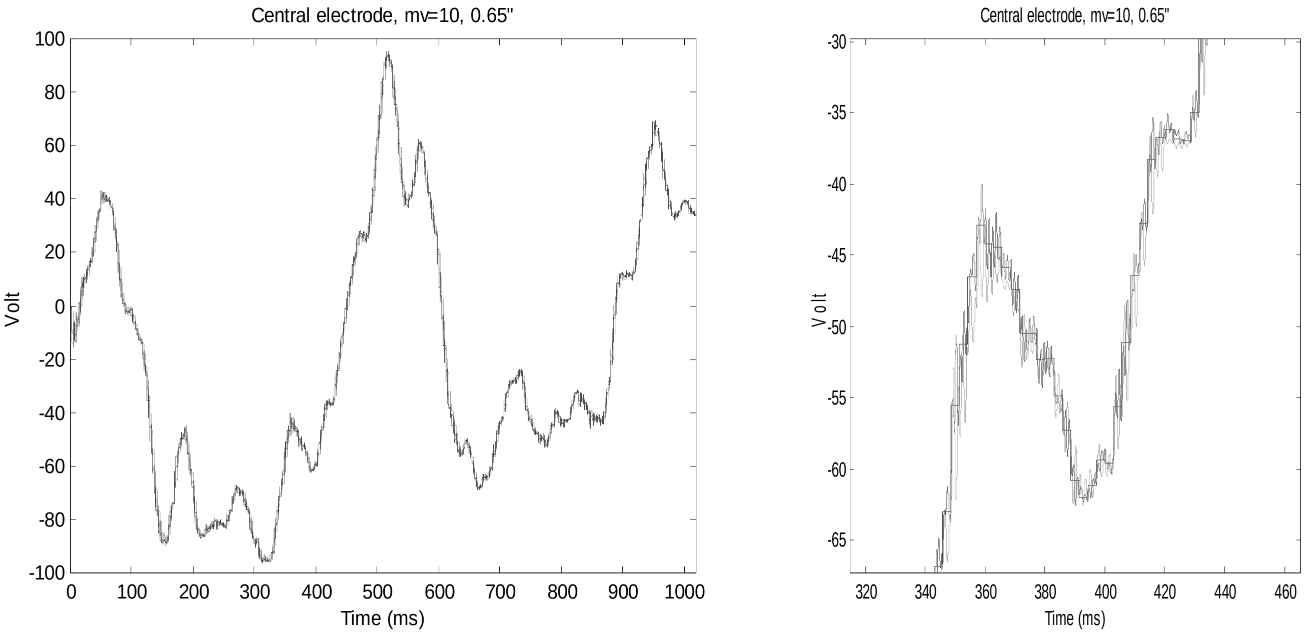


Figure 8-1: Evolution of the potential of the central electrode compensating for a 0.65" seeing for a star of mv=10

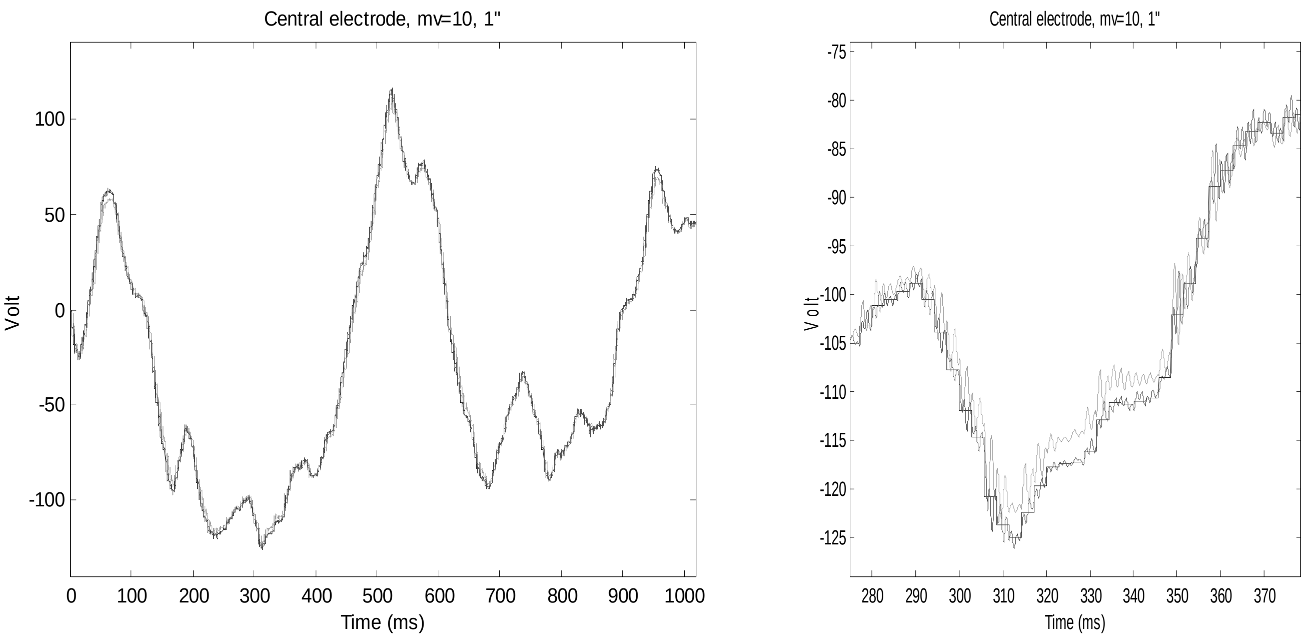


Figure 8-2: Evolution of the potential of the central electrode compensating for a 1" seeing for a star of mv=10

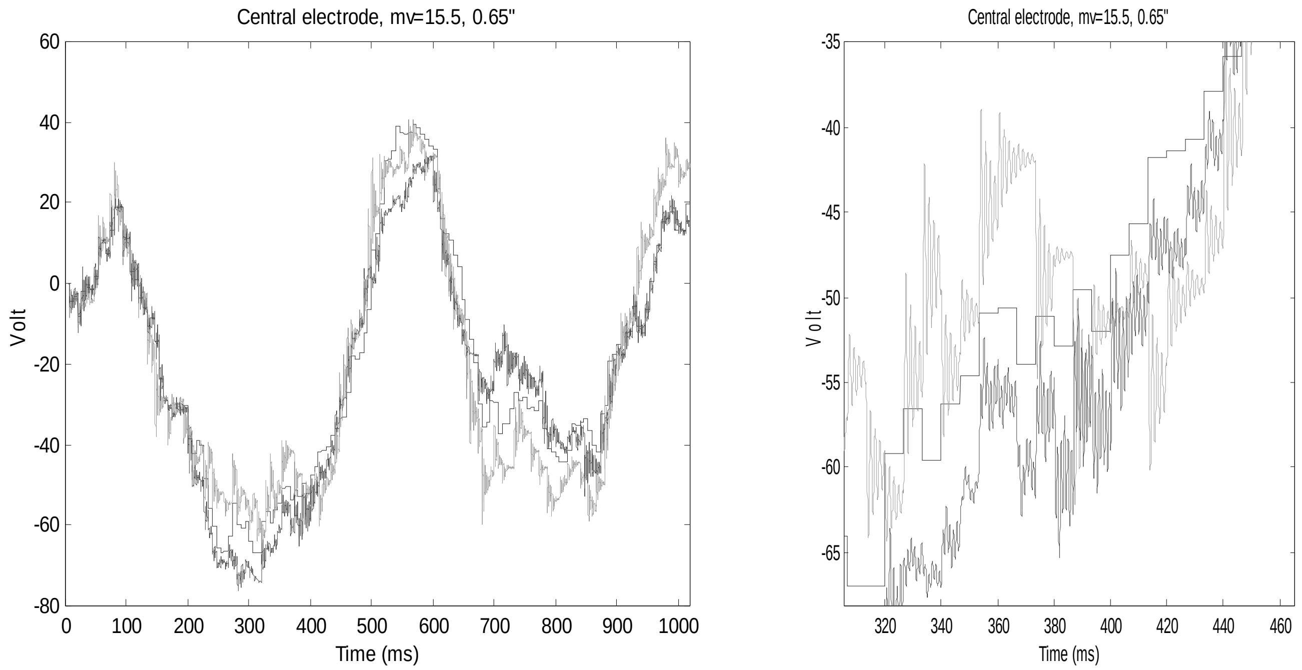


Figure 8-3: Evolution of the potential of the central electrode compensating for a 0.65" seeing for a star of mv=15.5

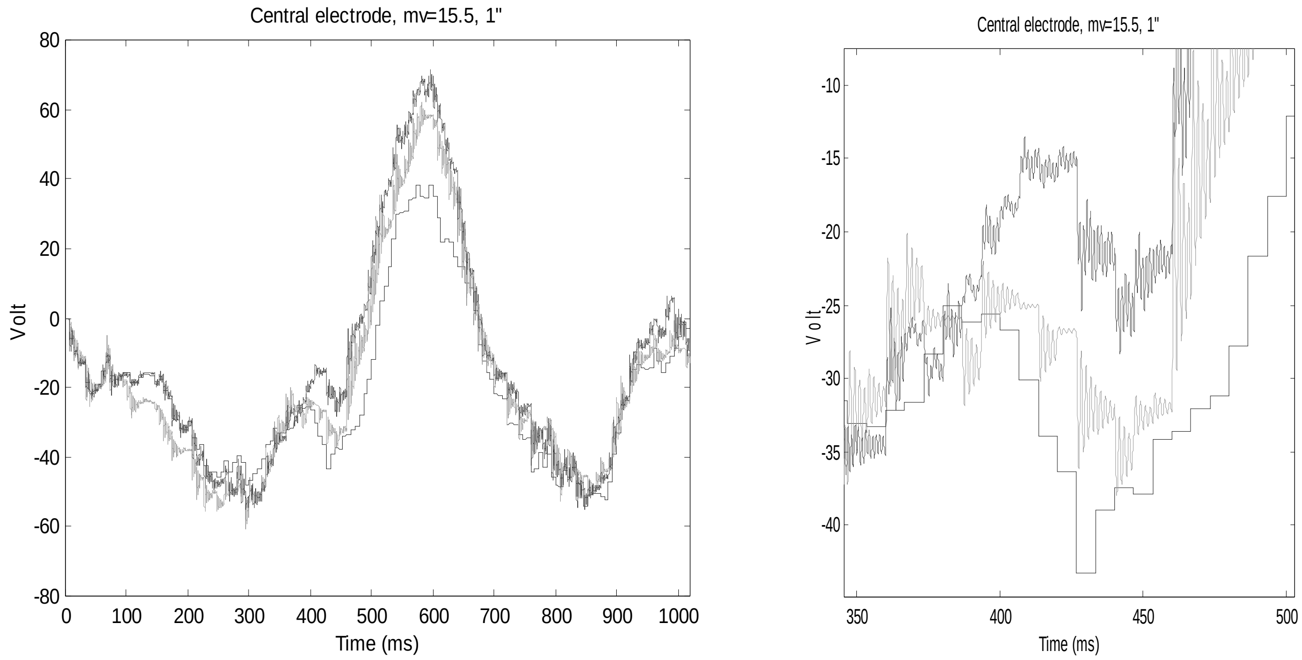


Figure 8-4: Evolution of the potential of the central electrode compensating for a 1" seeing for a star of mv=15.5

8.2 Strehl Loss

We ran several simulations to understand the impact of the dynamic of the mirror on the resulting Strehl ratio. In the case of a bright star simulations show that the Strehl loss is around 1% to 1.5% (absolute) and that the simulation that simulated the complete mirror dynamics (red line) gave better results compared to the simplified dynamics.

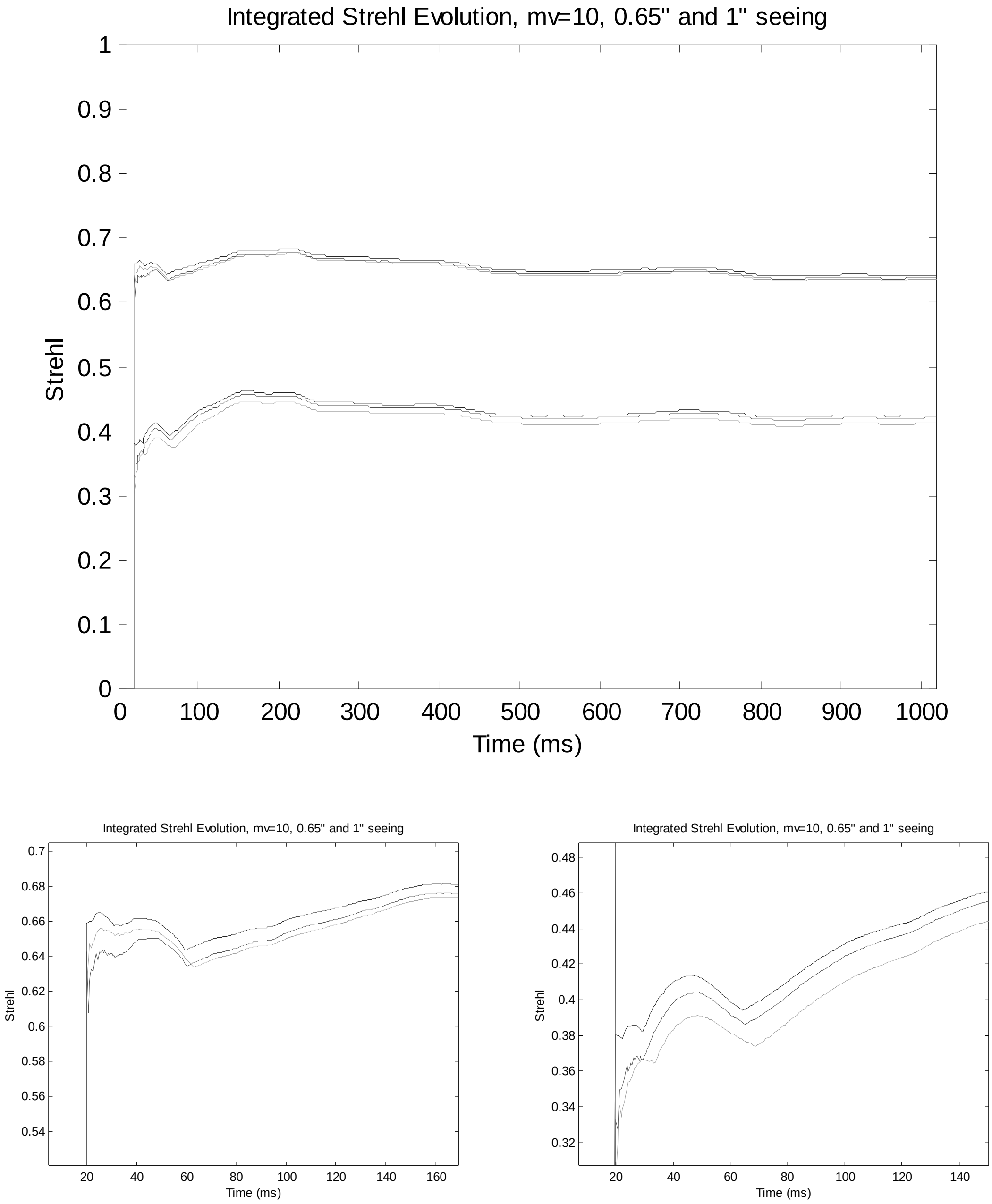


Figure 5: Strehl ratio for a guide star of mv=10, in case of 0.65" and 1" seeing without mirror dynamics (blue line), with simplified dynamics (green line) and full dynamics (red line).

In case of faint star we have a bigger effect which in some cases can turn to be an improvement. In general the difference between the ideal case and the real case is still around 1% to 2% (absolute).

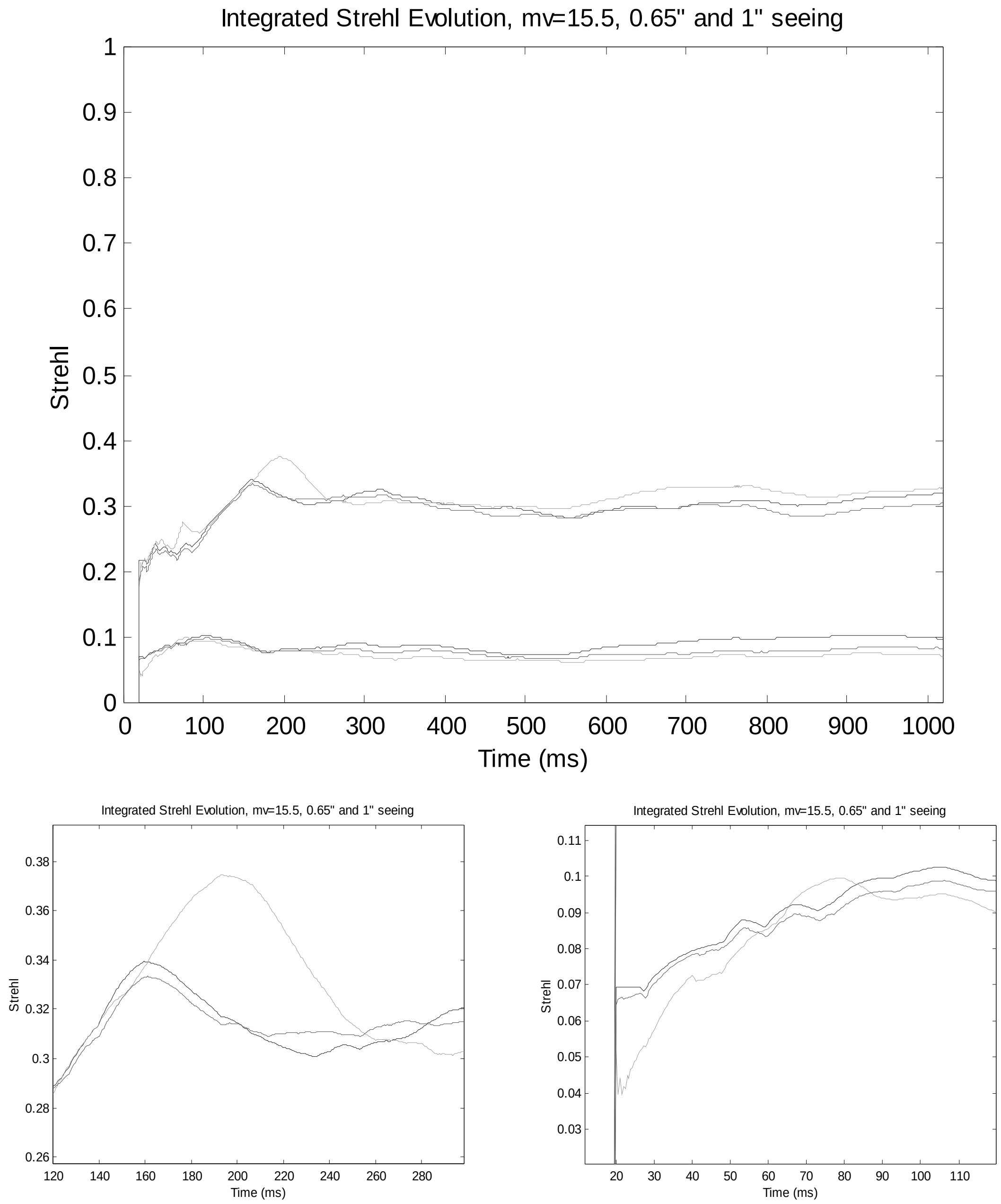


Figure 6: Strehl ratio for a guide star of mv=15.5, in case of 0.65" and 1" seeing without mirror dynamics (blue line), with simplified dynamics (green line) and full dynamics (red line).

8.3 Piston Effect

Given the fact that the entire mirror oscillates due to the effect of the resonant modes, an additional piston could be introduced in the light path at the frequency of the resonant modes.
We computed the piston introduced into the system in the previous cases computing the average position of the mirror in every sample of the simulation given the history of the voltages computed by the control loop. The simulation ran without any piston removal algorithm so we introduced this algorithm during this computation in order to compute only the piston due to the resonant modes. It is possible to do so because the piston removal algorithm acts purely in open loop[13] thus the values produced by the control loop are not affected by the presence of the piston removal algorithm. This algorithm is described in [RD 2].

The following table shows the piston contribution of the DM. The numbers are RMS value over windows of 48ms (first number) and 290ms (second number) in the different conditions analysed. These results show that there is no piston contribution due to the resonance. The introduction of the complete dynamics raises the piston contribution by several orders of magnitude, but still very small.

Magnitude	Seeing	Dynamics		
		No	Simple	Complete
10	0.65”	6e-21/6e-21 [m]	18e-21/15e-21 [m]	15e-15/12e-15 [m]
10	1”	2e-21/4e-21 [m]	15e-21/13e-21 [m]	15e-15/12e-15 [m]
15.5	0.65”	6e-21/7e-21 [m]	15e-21/13e-21 [m]	20e-15/18e-15 [m]
15.5	1”	3e-21/4e-21 [m]	20e-21/18e-21 [m]	18e-15/15e-15 [m]

Disabling the piston removal algorithm and computing again for some of the previous cases we get:

Magnitude	Seeing	Dynamics		
		No	Simple	Complete
10	0.65”	200 / 450 [nm]	200 / 450 [nm]	200 / 450 [nm]

that clarifies the importance of the piston removal algorithm. Moreover, it is worth noting that without the piston removal algorithm the amount of piston introduced in closed loop is independent from the particular model of the mirror, so the piston introduced by the dynamics, which has to be present, is neglectable compared to these numbers.

The piston removal algorithm is a two-steps process: in the first step the command-space piston is removed, in the second the mirror-space piston is removed. The first is totally internal to the controller: in fact the piston here is defined as one of the eigenvector of the interaction matrix. A change of the mirror properties will change the eigenvectors and thus the command-space piston but a simple recomputation of the interaction matrix will re-tune the system properly.
The second step is external because the mirror influence functions are used, which have been computed with some additional equipment. If the mirror properties change the system cannot be re-tuned as easily as

[13] The piston removal algorithm moves the DM using the piston mode that is invisible to the sensor. The sensor measures are not affected by the piston mode of the DM thus the piston removal algorithm can freely reconfigure the DM through the piston mode without changing the sensor measures by any means, so without changing the evolution of the loop and its stability properties.

recomputing the interaction matrix. It is of interest to compute the effectiveness of the piston removal algorithm when the influence function are not know with high accuracy as in the first case. If we deliberately add a uniform random noise of +- 6% on the knowledge of the influence function we get:

Magnitude	Seeing	Dynamics		
		No	Simple	Complete
10	0.65”	70/160 [nm]	70/160 [nm]	25/35 [nm]

This can account of a change of the influence functions or a limitation in the precision of the measurement. The interesting thing is that the simulation with the complete mirror dynamics is less affected, in terms of piston introduction, by this error in the knowledge of the influence functions, which would suggest not to try to compensate for the resonant modes. More investigation on this fact is required.

Another thing to note is that an error of 6% in the knowledge of the influence function can make the piston removal algorithm totally ineffective (compare with the previous table).

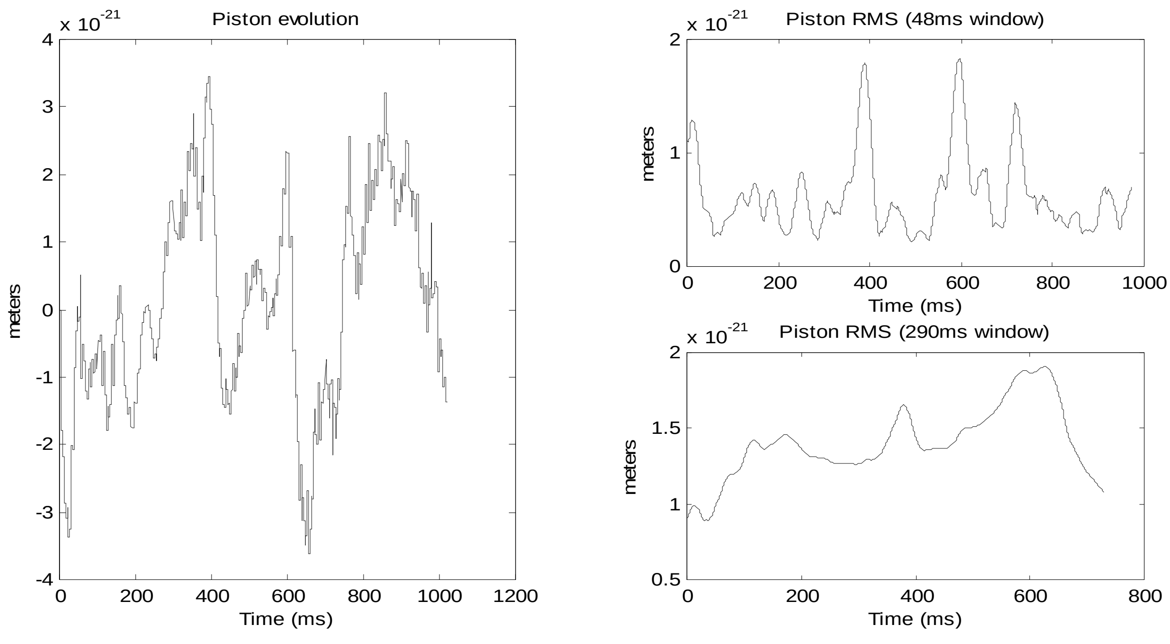


Figure 7: Piston present on the mirror in the ideal case

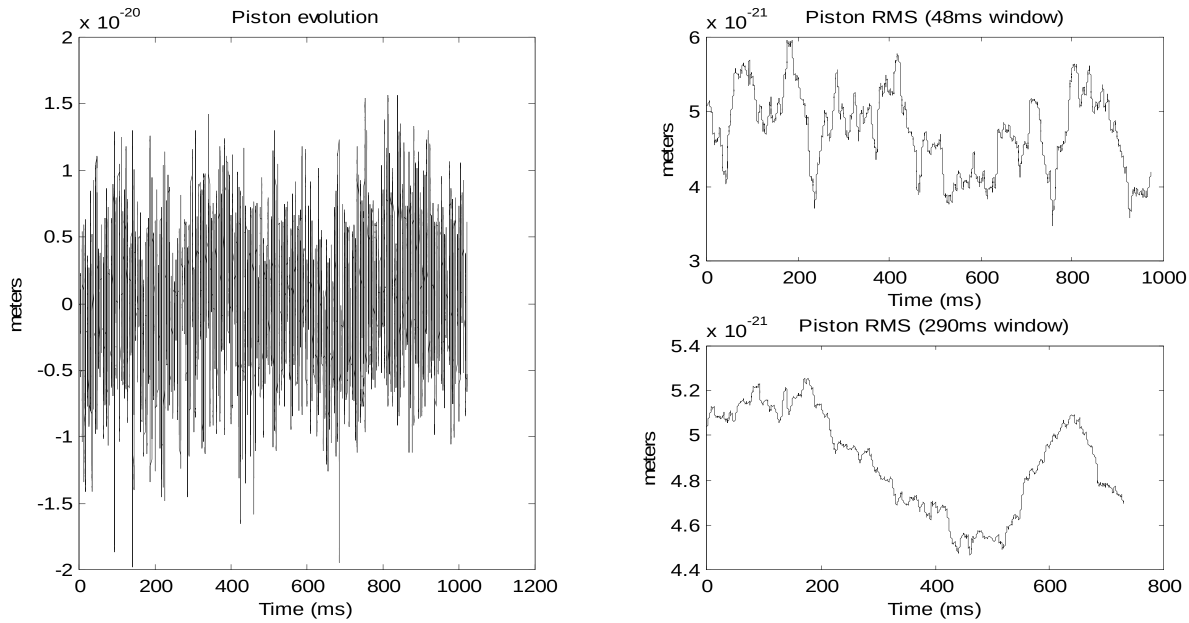


Figure 8: Piston present on the mirror in the simplified case

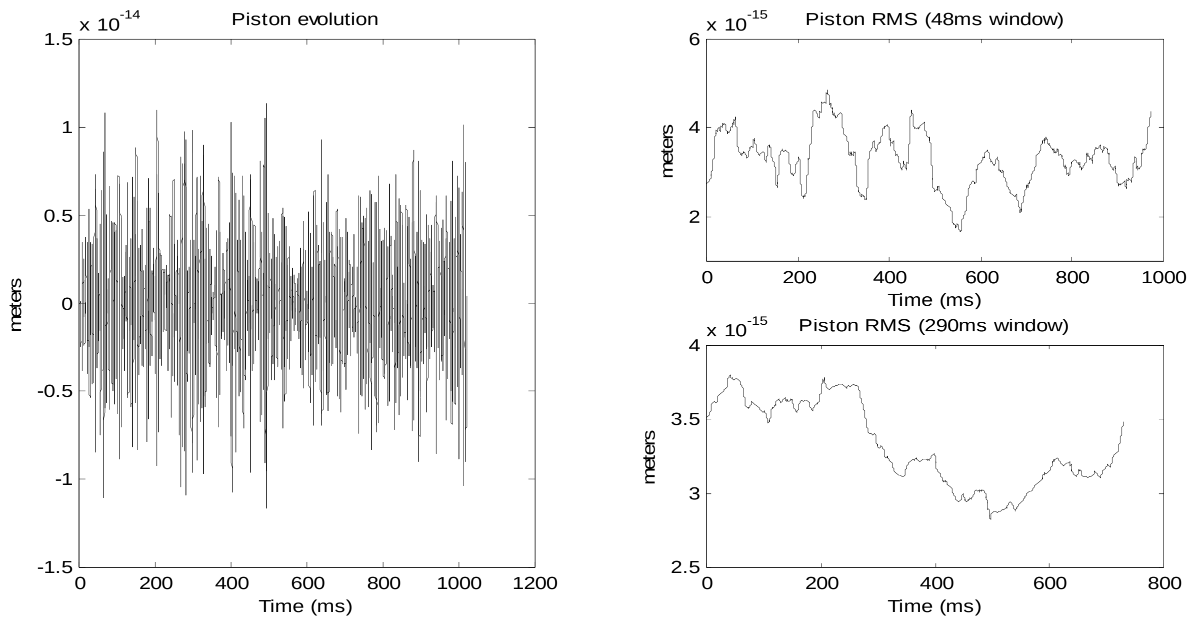


Figure 9: Piston present on the mirror in the real case

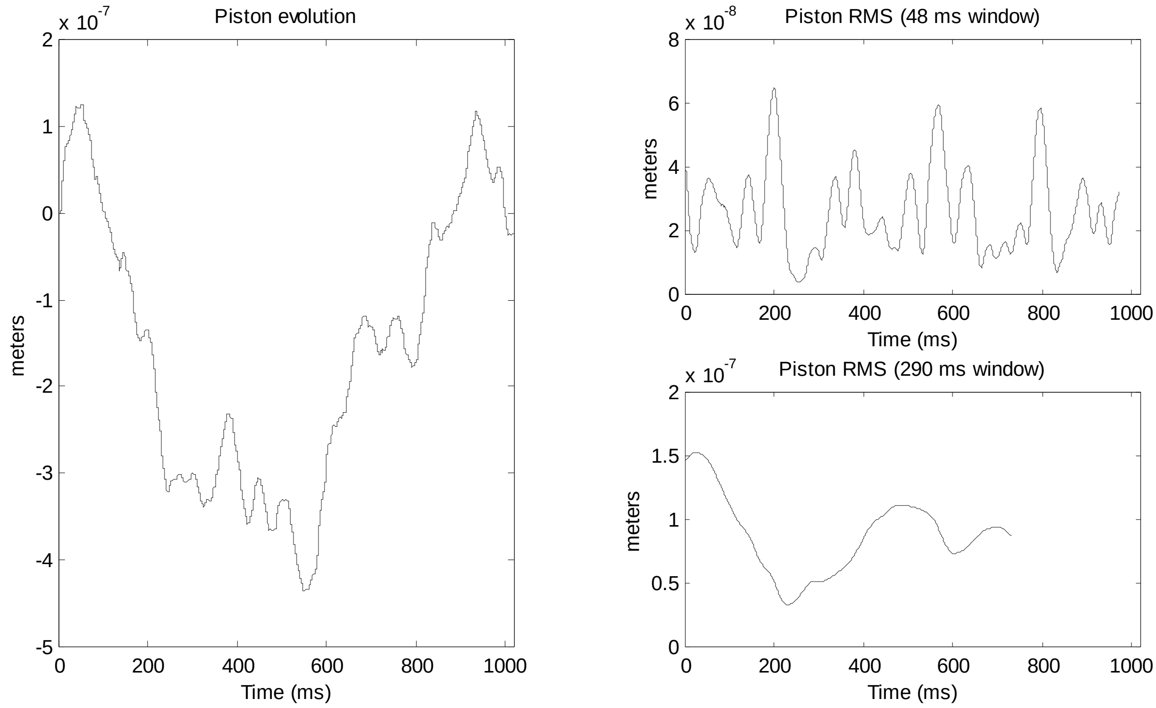


Figure 10: Piston present on the mirror without dynamics with 10% in the influence functions

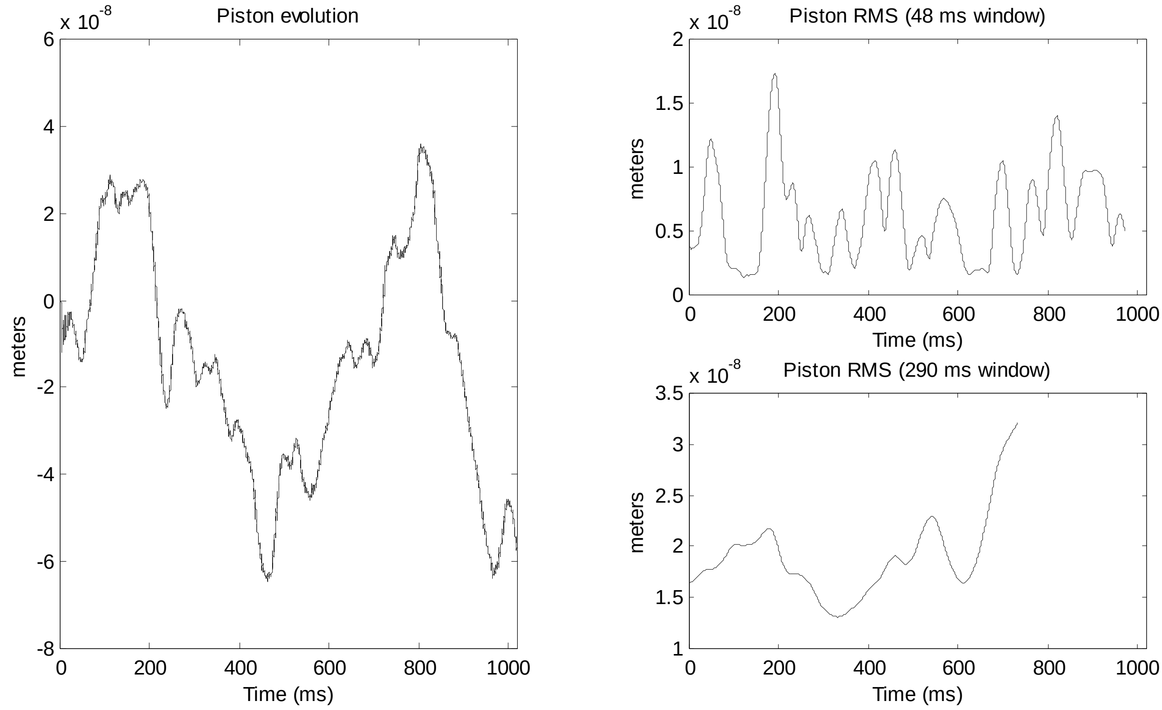


Figure 11: Piston present on the mirror in the real case with 10% in the influence functions

9 Shaping Filter Implementation

To implement the shaping filter a new development is needed. A simple IIR digital filter (4^{th} order, all-pass) has to be implemented either in the RTC or in the HVA electronics. In any case, the DAC has to run at high speed; in our simulations we ran it at 33.6 kHz but this may turn out to be a too much demanding requirement. We analysed the effect of reducing the sampling rate of the shaping filter in section 7.2.

If the digital filter is implemented into the RTC it will be required to have a communication link from the RTC to the HVA running at 33.6 kHz. Each sample is composed by 62 values of 2 bytes each which gives a transfer rate of more than 4 MB/s, which is not impossible but demanding, in particular for the RTC which has also other things to do.
The simplest solution is to place this digital filter in the HVA electronics. The filter has to carry in front a holder which can operate at variable frequency in order to let the RTC run at the desired frequency (350 Hz max for VLTI, 500 Hz max for SINFONI). This holder will produce a signal at 33.600 kHz that will feed the digital filter and in turn the DAC.

9.1 Shaping Filter in the RTC

The shaping filter can be implemented into the RTC.
Since the RTC is structured with multiple tasks running at the same time, it is not possible to devote the whole processor to implement the filter and thus it becomes crucial to create a proper synchronisation mechanism in order to generate the data at a constant rate.
The required sampling rate is quite high to be able to use the internal timers as synchronising mechanism, so the obvious solution is to use the TIM board to generate the required interrupts regularly in order to run the filter with high temporal precision.

A simple non-optimised code that implements the desired 3^{rd} order IIR digital filter is the following:

```
extern "C" void IIRfilter()
{
  float *wn  = w[0];
  float *wn1 = w[1];
  float *wn2 = w[2];
  float *wn3 = w[3];

    // Compute w[n]
  for(int k=0;k<NumChannels;k++) {
    wn[k]=a[0]*wn1[k]+a[1]*wn2[k]+a[2]*wn3[k]+x[k];
  }

  // Compute and output y[n]
  float y;
  for(int k=0;k<NumChannels;k++) {
    y=b[0]*wn[k]+b[1]*wn1[k]+b[2]*wn2[k]+b[3]*wn3[k];

    // output y
    output[k]= y;

  }

  // update status
  float *tmp = w[0];
  w[0]=w[3];
  w[3]=w[2];
  w[2]=w[1];
  w[1]=tmp;

}
```

This routine is attached to the TIM board through a user-level semaphore or directly invoked from an ISR.

Table 9-1has been computed using a Motorola PPC running at 400 MHz.
These performance figures have been obtained in the following way: the filter was running under task A for a certain number of cycles. At the same time task B is doing a very simple job repeatedly. The two task are synchronised so it is possible to count the number of cycles task B run when task A was active at different frequency or not active. The ratio gives the overall computing power that the IIR + TIM board interaction is taking
Column Frequency gives the frequency at which the IIR filter was running, 0 Hz means the filter is not running. Column “Available CPU time” gives the percentage of time available for all the other activities. Column “Task A FRQ” gives the effective frequency at which the IIR filter is running, measured with the TIM board itself. Column “Task B FRQ” gives the frequency at which the concurrent task was able to run. The column “ISR/User level” indicates if the IIR filter was running at user level or interrupt level.

Frequency	Available CPU time	Task A FRQ	Task B FRQ	Task A: ISR/User level
0 Hz	100 %	N/A	31135072	N/A
0 Hz		N/A	30484744	N/A
14.2 kHz	15%	13803	4793004	User level
14.2 kHz	15%	13823	4746701	User level
14.2 kHz	15%	13812	4778127	User level
10 kHz	41%	9988	12642691	User level
10 kHz	41%	9992	12631052	User level
10 kHz	39%	9985	12125280	User level
8.3 kHz	51%	8306	15660185	User level
8.3 kHz	49%	8326	15050792	User level
8.3 kHz	49%	8313	15097197	User level

Table 9-1: Computing power required to run the IIR filter

Although the filter was not optimised it seems very difficult to constrain its computing power consumption below 20% to leave at least 80% to the other, crucial tasks. In any case the pure IIR computational time is much less of what we found here [compute it] so chances to change this picture purely optimising the filter implementation (i.e.: implementing it in assembler) are very low.

9.2 Shaping Filter in the HVA

Thanks to the modularity of the system the shaping filter can be implemented in HVA rack. Moreover it follows the ESO standards and it uses normal ESO equipment: this can help to implement the filter in the future without designing for it in advance.
The filter can be implemented in two ways:

- As a process in the HVA LCU board
- As a separate device (daughter-board of the HVA LCU or additional board)

The first solution suffer from similar problems as the solution with the RTC, but here the 50% of the computing power may be enough to complete all the tasks the HVA LCU has to accomplish that are much simpler than the tasks of the RTC. In any case a TIM board is required to give this process the correct pace.

The best solution is an additional device that contains a simple DSP that is very efficient in implementing digital filters. No additional TIM board is required since the DSP itself can give the pace to the computation since a mono-task design is possible.

10 Conclusions

We analysed the problem of the resonant mirror modes. We divided the effect of this problem into image quality degradation (Strehl loss) and piston fluctuation.

In the following table we recall the results we found analysing the Strehl degradation due to the resonant mirror modes. We compute the Strehl degradation factor and we compute which is the equivalent phase lag to obtain the same degradation factor.
To solve this problem one can design a shaping filter, which will act in open-loop simply compensating for the undesired mirror dynamics.
Implementing the shaping filter in the RTC will cause the control loop to run slower. Currently the control loop is designed to complete one cycle in 0.5ms, given the full computer power. The shaping filter implemented in the RTC will take half of the computer power, doubling the time required to complete one cycle. At least for the first three cases in the table, the benefit of the shaping filter will be destroyed by the additional phase lag which is exactly in the same order.

Corrected Strehl	Absolute Degradation	Relative Degradation	Degradation Factor	Equivalent phase lag
65%	1.5%	2.3%	97.7	0.4ms
45%	1.5%	3.3%	96.7	0.5ms
30%	1.5%	5%	95	0.7ms
10%	1.5%	15%	85	1.3ms

This means that if the shaping filter is really necessary, it cannot be implemented in the Real Time Computer.
The only reasonable solution is to put this filter into the HVA electronics and the most promising solution seems to be an additional computational module.
But we've also seen that such a filter is very sensitive to a correct tuning (during installation) and to the constancy of the mirror properties potentially leading to worse results in case of bad tuning.

The filter could be tuned in-place turning on the compensation one channel at a time and changing its parameters in order to get the best results, but this seems long and complicated.

On the second effect, the piston, we discovered that there is no effect at all (it is neglectable) so no counter-measures are suggested.
The problem here is on another topic which doesn't cover the resonant mirror modes but, instead, the stability and measurement accuracy of the mirror influence function, the starting point to build up an effective piston removal algorithm.

So the conclusion is:

- If a loss of 1.5% of Strehl is acceptable, nothing should be done for the problem of the resonant mirror modes.

- If case this will turn out to be a problem, the proposed solution is an additional computational module to be installed on the HVA LCU which will implement a simple (maximum 4th order IIR filter) shaping filter.

Further topics that need study and attention:

In case a shaping filter is required:

- Performance of the system with a shaping filter, tuned and de-tuned.
- Design of the tools and procedure to tune the filter.

On a different topic:

- Performance of the piston removal algorithm function of the knowledge of the influence functions and the piston mode.
- Performance of the piston removal algorithm in presence of the shaping filter.

11 Appendix

The following is the base64 encoding (uuencode) of the archive of all the Matlab models used in this work. To retrieve the file simply scan this and the following pages, use a OCR software to get the text back and decode it with uudecode.


```
begin 640 Models.zip
M4$L#!!0‘‘‘‘(‘/U.E"G%>/‘5XP$‘‘+‘‘‘‘‘3‘‘‘‘0DE--C!S:&%P:6YG,E]Z
M+FUA=/-U#/%Q=%(PU3-0\’4,T4W+S$G540C(22Q)RR_*M5((<‘[W]--1<"Y*
M32Q)35’(S[-2"‘?2+JG)"D8&"@:65J:F5J;F0+:!@0+)@(’1TY>/@8%!@YV!
M@0U(<P‘Q,P,$L$+YC$B8’XB=/’W-#(HS$@LR\]*-XJN‘XLP,50790/4L#‘I0
M=0N‘N(H!/R@@()]-0#XLL2@S,2DG%9=\3DDF+BDP‘/E[!0/"WXQ0<6S^AJFO
M0%+/QH&JG@5-/2<0*P"QR]\32O(9;_9#A1E:?C$K72M[;P^C8>K0P>L+R0L6
M6ARVA]+[Z>7>SV=_UI]X]![NWB73F4V/E;ZRA]&XW+M6Z?@D+JT#]E‘:[%X+
M!B3[H>IPN9<3*O[QN6)?X’L&!Y!^‘R3]+‘3TLS‘P@=,<2%\$"W’IF0%,,X/3
M"G+Z30‘2(<4,>(%G7D%IB4MJ3F(E=GG_TA*\"C+SP7*^B25%F158Y,’F^R7F
MXDK@$/-Q*P#K=R_*+\61T2#Z<2OPRR])Q1<$H<6I12Z))8FXY,-2BXHS\_-P
M23,0GSZ8P’$!2Q]O>F:WJIG^MT-/’X32%Q.4-1#Z$I#TP>3QY5M\Z1X&$.D>
MH2^!@3[V:##@M@<$F!A0[:&V>@8&W/Y‘CQ=T]4Q$ZL,6GR"]‘%!+‘P04‘‘‘‘
M"‘#;B90I^UJ+JJD!‘‘!(!P‘‘$@‘‘‘$))338P<VAA<&EN9U]S+FUA=,54S4["
M0!‘>2D6X&,^>]L@!35%10V*4GT2;4.3‘SU%67;2AM*0M"?(2<C=1’\"’X.BC
M\‘1Z\(#;OU":;L&#..EVV_V^;Z:S,UNI4*\4BBBW)R"I4-_MR‘K)H)J"S8ZF
M]_*H5FJ)U0PJZ02;Y‘YI:AZUZ%PFMVA?0-GC?/8PGSN@SX*‘?FT0$Z4M‘)@D
M‘.@%23KBX-B&^Q[S#8M;%*4CP7C‘?5F]OS:L]3B,^EW*YP&YO#<Z1A!M_25X
M=PG>Q+J,;Q3"PA539D&V6;D,89YWS%T/R]OC7_KXB>0BGPOPXW0_*K/*+$6?
MM^GX>MIY27^^G[OS9!WQQ]/Q-"K^B=\?+/H+QD^YZQ^;I\^-U^\S2R_X]/P2
M/4^_D+:+G4>-7ZW?‘)S^LFKI[Z\VO=4-B#11[0_,,E’P8SA^-3‘C";)F8Q(V
M=7D8@MO^J[C’:D#’/YM@ZR]T;<‘X"(Z>3:AJ)HG:@H9!]#(V,0MO$MV0-94%
M0["^[/[@[%IP;L56USGVG[JV3^?A4><OJM\]F_?[7->&]<1)‘SN.9=X_XJ_X
M‘.P\@G4)\KD5=6’UM+0_4$L#!!0‘‘‘‘(‘*2)E"FGHRGXW@$‘‘)‘‘‘‘2‘‘‘‘
M0DE--C!S:&%P:6YG7WHN;6%T\W4,\7%T4C#5,U#P=0S13<O,2=51",A)+$G+
M+\JU4@AP#O?TTU%P+DI-+$E-4<C/LU((!](NJ<D*1@8*AN96AL96!F9‘MH&!
M‘LF‘@=’3EX^!@8&#G8&!#40#,3,#!+!"^8Q(&*36R=/7S*‘X([$@,R\]O@HD
MSLQ059‘-5,_"H‘!5MP"(JQCP@P("\MD$Y,,2BS(3DW)2<<GGE&3BD@(#D%]F
M,"#\S0@5Q^9OF/H,)/5L’*CJF=’4<P*Q!!"[_#VA))_Q9G]/K4S6AD7O[6$T
M3!X&.C1.VD1S[K&’TOOIY3Z’%Z(<RV>]LH?1G\_^K#_QZ/U^F’R?Q;,K-R=O
MMX?2^QF0[+-‘MH\!U3YT]W%"Q;?R,-<V27‘[@/0;(.EG(:"?A8$)G*9‘^B)8
MB$NO#‘R0]‘E*"\CI,P%(A!0SX‘6>>06E)2ZI.8F5V.7]2TOP*LC,!\OY)I84
M959@D0>;[Y>8BRL!0\S’K0"LW[THOQ1’1H+HQZW‘+[\D%5\0A!:G%KDDEB3B
MD@]++2K.S,_#)4U"^F‘"QP4L?;SIF=VJ9OK?#CU]$$I?3%#60.A+0-(’D\>7
M3_&E>QA‘I’N$O@0&^MBCP8#;’A!@8D"UA]KJ&1AP^P,]7M#5,Q&I#UM\@O0"
M‘%!+‘P04‘‘‘‘"‘‘1BI0I6VT]A\@!‘‘!P!P‘‘$P‘‘‘$))338P<VAA<&EN9U]Z
M."YM873S=0SQ<712,-4S4/!U#-%-R\Q)U5$(R$DL2<LORK52"’‘.]_3347‘N
M2DTL24U1R,^S4@@’TBZIR0I&!@J&YE:&9E;&1D"V@8$"R8"!T=.7CX&!X04;
M‘P,0,7‘‘,3,#!+!"^8Q(F!^(G3Q]S0R*,Q(+,O/2XZLL@.+,#%4%V4#U+‘P*
M4’4+@+B*‘3\H(""?34‘^++$H,S$I)Q67?$Y))BXI,‘#YNX,!X6]&J#@V?\/4
M1R"I9^-‘5<^$IIX3B‘6‘V.A‘X?3#QJ_M831,7/O[Y3_ULR[:0^G]]’+/L_‘V
M)_V0._8P&B9^NNE=SZ’#I^RA--@]%LCF,Z":C^X>3JCX"7;;V:’S?]N#]!L@
MZ6<AH)\%Z&)0F@’[BX6X],@‘IIG!<8V<_A*‘1$@Q‘U[@F5=06N*2FI-8B5W>
MO[0$KX+,?+"<;V))468%%GFP^7Z)N;@2*,1\W‘K‘^MV+\DMQ9!2(?MP*_/)+
M4O$%06AQ:I%+8DDB+OFPU*+BS/P\7-(,Q*</)G!<P-+’FY[9K6JF\ACI@U#Z
```

M8H*R!D)?‘I(^F#R^?(DOW<,‘(MTC]"4PT,<>#0;<]H‘‘K.R@E7H&!MS^0(\7
M=/5,1.K#%I\@OO!02P,$%‘‘‘‘‘@‘JG"3*::K)3N<‘0‘‘*‘<‘‘‘X‘‘‘!"24TV
M,’-I;5]S+FUA=,54RT["0!2]+96@"^/:U2Q=H"E&3.Q*’HDVH<BBXM*..B0-
MA9)V2)"?\‘\,’^‘G&.-7Z1*G#]*Q=BA=J#>]?9US.G/GGH[1,#N-)JH?J<AH
MF(<#VR%5U’,P’;C>2$.]UHW>K:*61S‘E#\@=:\B<$M0F]ZAVAFHGFEK7V,VQ
MJJJH<("D&[L‘L"@#L‘,J+$L0Q5;\+’&YP[*I&Z>J;X]N_8@G,<5\,F1\!5#,
M6["<P_J8Y.##’+R//1O?.42$.]0606$$=5N0U"W%[[/J7O%5CE].\2’%ESG=
M#(J-<\GQRY7O?#G%+[%U[RP[RVUVO\?R\VG_^>#CY3R^ON?-.SU^V,?7MQ_U
M*CDZA<TLL$2@ZRF;^0D@\D_0*]X_%CN9L;]$H8\G4]HF#G[,QJ^F="W!=D/,
MP-2S9QEX^/TN’HD,%GU?3‘CU%YX[%1@]THL)79>2=4MP[1.OC2D6X7WB^;8[
M%L&Y?D[Z)(>]D...%?73?^HL3I?X>[/_.^WW521^3W06_,TX!R‘>)XC5WO!;
M?‘!Q’5G[(<_G]\.B_0SF^0502P,$%‘‘‘‘‘@‘S’63*<=W]MB?‘0‘‘*‘<‘‘‘\‘
M‘‘!"24TV,’-I;5]S,BYM873%5,M.@T‘4O5!L:DR,:U>S=%$-);6)K.PCL212
MN\"ZE%&G"2DM#8^D]B?L%YA^@!_AI[CL%^C6X=&‘A(&R4&^XO,XYW)FYAU’;
MVG6[@\[/1*2VM=.Q89(Z&IK8’5OV5$;#[ITRJ*.N3;!+GI‘UDY’F$=0CCZAQ
M@1I-N=F2)0E)HBBBT@&<HAX"P+H*0‘^HT:Q‘&’O1,Y?(‘YH=16V)CC&]=Z2‘
MQU’%<CZA?‘%0Q%O37$)^S‘OP20$^PK:!’TS"PDW78$%!^//6(9XW%[W/FO>6
M+R;XU10?4GP^H5M‘N3K]!+]:^\GG4_P*7??59K79I_=’-+]>CE]//M\NH^M[
MT;C3]?T^?O05+JT3"G0"’9D#X?B’PFY^‘@C]X_<JZ1^=GC0’<D.9S3VW1TS\
MG(W?>&XNP;‘"3,6N;2PR\.#[‘SQE&2S\/IL0Z*]LRV,8/=2S"0/+)7E+<.L0
MNX==S,)’Q’8,:\:""_T<]XD/>L%’’2OKI__4Z0E=[._=_N^TW[<1^SW6Z?‘W
M=4Z‘7<>/[=[P6WP‘]CRR]L,D/[D?ENVG/\YO4$L#!!0‘‘‘‘(‘%UQDRD8^64:
MS0$‘‘&‘‘‘‘‘.‘‘‘‘0DE--C!S:6U?>BYM873S=0SQ<712,-4S4/!U#-%-R\Q)
MU5$(R$DL2<LORK52"’‘.]_3347‘N2DTL24U1R,^S4@@I355P24U6,+14,#2Q
M,C2P,C57,#(P,%‘@&3‘P>OKR,3‘PW&!C8‘‘B!@X@9F:‘‘%8HGQ$)<P&QDZ>O
MF4%Q9FY\%40=(U!’54$V4#T+@P)4W0(@ADKC!‘4$Y+,)R(<E%F4F)N6DXI+/
M*<G$)04&(’^#W‘#S-R-4’)N_8>H=D-2SH:EG0E//"<0"0/SY[,_Z$X_>[V=‘
M,J>#@31[(Y#4LW&@JL=E;T^M3-:&1>_M831,O$/CI$TTYQY[*+T?9+X%‘VY_
MH;N’$RJN<=/I3?ZD,GMT_2Q8]+,@Z6>!BE<QQ#’H,AA"_,="7/IC8("D-U#<
M(J>W!"‘14LR‘%WCF%926N*3F)%9BE_<O+<&K(#,?+.>;6%*468%%’FR^7V(N
MK@0),1^W‘K!^]Z+\4AP9‘Z(?MP*__))4?$$06IQ:Y))8DHA+/BRUJ#@S/P^7
M-‘/QZ80)’!>P=/*F9W:KFNE_.Y!^‘Z+T0P‘3E#40^A*0]!&3/Y’M04__,("<
M_F’Z$ACH8X\&‘VY[0‘!6AM!*/0,#;G^@QPNZ>B8B]6&+3Y‘[‘5!+‘P04‘‘‘‘
M"‘!S=I,I3B3#VL\!‘‘!@!P‘‘#P‘‘‘$))338P<VEM7WHR+FUA=/-U#/%Q=%(P
MU3-0\’4,T4W+S$G540C(22Q)RR_*M5((<‘[W]--1<"Y*32Q)35’(S[-2""E-
M57!)358PM%0P-+$R-;0R-E,P,C‘P4"‘9,#!Z^O(Q,##<8&-@‘"(&#B!F9H‘‘
M5BB?$0ES‘[&3IZ^907%F;GR5$5@=(U!’54$V4#T+@P)4W0(@KF+‘#PH(R&<3
MD‘]++,I,3,I)Q26?4Y*)2PH,0/X&N0’F;T:H.#9_P]0[(*EG0U//A*:>$X@%
M@-CE[PDE^8PW^QF0S.E@(,W>""3U;!RHZG’9Z_!"E&/YK%?V,!HFWF?Q[,K-
MR=OMH?1^D/D6#+C]A>X>3JBX0&F<WPV9R?;H^EFPZ&=!TL\"%:]BB&/093"$
M^(^%N/3’P‘!);Z"X14YO"4‘BI)@!+_#,*R@M<4G-2:S$+N]?6H)7068^6,XW
L:0HLP*+/-A\O\1<7‘D28CYN!6#][D7YI3@R!D0_;@5^^26I^((@M#BUR"6Q
M)!&7?%AJ47%F?AXN:0;BTPD3."Y@Z>1-S^Q6-=/_=B#]!D3IAP‘F*&L@]"4@
MZ2,F?R+;@Y[^80‘Y_</T)3#0QQX-!MSV@‘"L#*&5>@8&W/Y‘CQ=T]4Q$ZL,6
MGR!W‘@!02P,$%‘‘‘‘‘@‘#4F4*=.2IS$,‘@‘‘R‘<‘‘‘T‘‘‘!"24TV,’1F7W,N
M;6%T\W4,\7%T4C#5,U#P=0S13<O,2=51",A)+$G++\JU4@AP#O?TTU%P+DI-
M+$E-4<C/LU((!](NJ<D*1@8*!I96!A961B9‘MH&!‘LF‘@=’3EX^!@<&!G8&!
M#4AS‘#$S‘P2P0OF,2)@3B)T\?<T,2M+BBZ’J&($ZJ@JR@>I9&!2@ZA8‘<14#
M?E!‘0#Z;@’Q88E%F8E).*B[YG)),7%)@‘/+W"@:$OQFAXMC\#5-?@:2>C0-5
M/0N:>E!8*0#Q+BN;/G_WD‘,P^OLVY=^2=W/A-$S=ZT\7I_ZVF.L‘I0](?-VL
M]/C,8@<H?0!D_PD&TMP[@P&W>]D8,-UK‘,0-"E6[LKN2#\#H‘.;VB%.+TN#T
ME5KNC-VS_.$T3!_G%*\"_>X9#E#Z‘)?!6</-BQ<X0.D#J]^J^R4^W.P‘I<’^
ML4!V’P.J^]#]PPD5#[+O6\YQ(L.1#VHO3#\+‘?TL#$P,H#0+TA?‘0EQZ9V"‘
MI&]06D).WPE‘(@26‘7‘‘S[R"TA*7U)S$2NSR_J4E>!5DYH/E?!-+BC(KL,B#

MS?=+S,65‘2#FXU8‘UN]>E%^*(R-"].-6X)=?DHHO"$*+4XM<$DL2<<F’I185
M9^;GX9)F0(]?W.F#"1P73-‘8(UX?!‘RDO@0D?3!Y?/D97WJ’‘41Z1^A+8*"/
M/1H,N.T!‘28&5’NHK9Z!‘;<_T.,%73T3D?JPQ2=(+P!02P,$%‘‘‘‘‘@‘^$J4
M*6S4FIT;‘@‘‘X‘<‘‘‘T‘‘‘!"24TV,’1F7WHN;6%T\W4,\7%T4C#5,U#P=0S1
M3<O,2=51",A)+$G++\JU4@AP#O?TTU%P+DI-+$E-4<C/LU((!](NJ<D*1@8*
M!I961L96)F9‘MH&!‘LF‘@=’3EX^!@2&"G8&!#4AS‘#$S‘P2P0OF,2)@3B)T\
M?<T,2M+BJZ#J&($ZJ@JR@>I9&!2@ZA8‘,4P>%R@@()]-0#XLL2@S,2DG%9=\
M3DDF+BDP‘/E[!P/"WXQ0<6S^AJGO0%+/QH&JGA5-/2BL-(!XSH%O^UY^>K]?
M=MO<T"OKWMO#Z"L)/7;3[R-HF’H8^!&Z>?I&]<WV4’K_!*%]MY.SUMA#Z?T@
M]YQ@(,W],QAPNY^-‘=/]!D‘LL5CSJ<#%]_8PVDMRQ919AQ&TYAIOP3^3$#1,
M7QU3SO_W%Y;;0^G]1[.F!"HH;K"’TOLK(A>72;L?MH?28/]8(+N/‘=5]Z/[A
MA(KG7GO6_ZBVW)X/:B],/PL!_2P,3.‘T"D[_+,2E?P8&2’H’I2WD])X‘)$**
M&?‘"S[R"TA*7U)S$2NSR_J4E>!5DYH/E?!-+BC(KL,B#S?=+S,65(2#FXU8‘
MUN]>E%^*(V-"].-6X)=?DHHO"$*+4XM<$DL2<<F’I1859^;GX9)F(#Y],(’C
M‘I8^WO3,;E4S_6^’GCX(I2\F*&L@]"4@Z8/)X\O7^-(]#"#2/4)?‘@-][-%@
MP&T/"#‘QH-I#;?4,#+C]@1XOZ.J9B-2’+3Y!>@%02P,$%‘‘‘‘‘@‘FWJ4*8-\
M<‘.‘‘0‘‘$‘<‘‘‘D‘‘‘!(5D%?<RYM873%5-U.@S‘4/F.XN!N?H9?&3,4E2PQW
MN"4Z(W,7<[N43KM(Q@8I7:)[@B7>^@‘^BH_AX]@""Q4IC,3H">6GW_?U<#@?
MM:W1C76!.B<&LJW1\<SU2‘L-/<QF/EV8:-B=]‘<MU*4$,_*(_*6))OS:(P^H
M;:"SCMDVS$Z;WQL&JAQ0Z]L’‘+!I‘/‘#]OFH0QQ[R7--&F+N:FS=A]%S’=;!
MG,_I@!+\G8\U%$=0@L]+\#&F+IYZ1(5[S%5!48AZ’4CKK27S>?5N^8;$;V3X
MD.%KDNX)JN4Y!W6>++^9S#??KH/3U\^/LO?,ZG7^ID?3G_7I.^A$_X5NJ._F
M&X#8+Z(WLE\<?AJ%4!C]9;!B/>+AEWS\=L4*":X?839FU’W.P:/U!WBA,E2\
MOIH0Z2^IOU(8.]:K"0.?D:)/<!<2VL,,J_‘QH:’K+U5PJ7_3/FE1+[2D8U7]
M])\Z1])M\5W_YZS?MY’Z/=4Y\#=Y#D&=1X0&W_/\-A]‘74?>_B?SY?VO:C^%
M]@M02P,$%‘‘‘‘‘@‘Q’J4*3$P#M25‘0‘‘(‘<‘‘‘D‘‘‘!(5D%?>BYM873S=0SQ
M<712,-4S4/!U#-%-R\Q)U5$(R$DL2<LORK52"’‘.]_3347‘N2DTL24U1R,^S
M4@@’TBZIR0I&!@J&IE9&1E8&YD"V@8$"R8"!T=.7CX&!808;‘P,0,7‘‘,3,#
M!+!"^8Q(&"3F$>887P7F,S-4%60#Q5@8%*#R"X"XB@$_*"‘@GTU‘/BRQ*#,Q
M*2<5EWQ.228N*3‘‘^3>!‘>%?1J@X-O_"U!L@J6=#4\^‘IIX)25\&‘VGV6##@
MM@==/2=4_(%?^NU#7O?LR=6_JSY<4NW6‘WMT?[(0T,\"]"DHKD’Z(EB(2S\,
M#)!T‘XHCY’23‘"1"BAGP‘L^\@M(2E]2<Q$KL\OZE)7@59.:#Y7P32XHR*[#(
M@\WW2\S%E;‘@YN-6‘-;O7I1?BB.!0_3C5N"77Y**+PA"BU.+7!)+$G’)AZ46
M%6?FY^&2)B%],8’C‘I8^WO3,;E4S_6]’*!^@QS,3E#40^A*0],’DB<W?Z.D>
M!A#I’J$O@8$^]F@PX+8’!)@84.VAMGH&!MS^P%8>(JM’+@])C4^0.P%02P,$
M%‘‘‘‘‘@‘‘$R4*;NEX+Z%‘‘‘‘Z‘‘‘‘‘H‘‘‘!S:6=N86PN;6%T\W4,\7%T4C#5
M,U#P=0S13<O,2=51",A)+$G++\JU4@AP#O?TTU%P+DI-+$E-4<C/LU((!](N
MJ<D*1@8*!I96QH96IA9‘MH&!‘LF‘@=’3EX^!@2$!B-F‘F‘-*@P‘KE,\(93-"
MY8HST_,2<Q@8.(%L#088^&‘_:R8(?+0W!H/’4/Y+>Y@\‘%!+‘0(4‘!0‘‘‘‘(
M‘/U.E"G%>/‘5XP$‘‘+‘’‘‘‘3‘‘‘‘‘‘‘‘‘‘‘‘(‘"V@0‘‘‘‘!"24TV,’-H87!I
M;F<R7WHN;6%T4$L!‘A0‘%‘‘‘‘‘@‘VXF4*?M:BZJI‘0‘‘2‘<‘‘!(‘‘‘‘‘‘‘‘‘
M‘‘‘@‘+:!%‘(‘‘$))338P<VAA<&EN9U]S+FUA=%!+‘0(4‘!0‘‘‘‘(‘*2)E"FG
MHRGXW@$‘‘)‘‘‘‘‘2‘‘‘‘‘‘‘‘‘‘‘‘(‘"V@>T#‘‘!"24TV,’-H87!I;F=?>BYM
M87102P$"%‘‘4‘‘‘‘"‘‘1BI0I6VT]A\@!‘‘!P!P‘‘$P‘‘‘‘‘‘‘‘‘‘‘‘"‘‘MH’[
M!0‘‘0DE--C!S:&%P:6YG7WHX+FUA=%!+‘0(4‘!0‘‘‘‘(‘*IPDRFFJR4[G‘$‘
M‘"@’‘‘‘.‘‘‘‘‘‘‘‘‘‘‘‘(‘"V@?0’‘‘!"24TV,’-I;5]S+FUA=%!+‘0(4‘!0‘
M‘‘‘(‘,QUDRG’=_;8GP$‘‘"@’‘‘‘/‘‘‘‘‘‘‘‘‘‘‘‘(‘"V@;P)‘‘!"24TV,’-I
M;5]S,BYM87102P$"%‘‘4‘‘‘‘"‘!=<9,I&/EE&LT!‘‘!@!P‘‘#@‘‘‘‘‘‘‘‘‘‘
M‘"‘‘MH&("P‘‘0DE--C!S:6U?>BYM87102P$"%‘‘4‘‘‘‘"‘!S=I,I3B3#VL\!
M‘‘!@!P‘‘#P‘‘‘‘‘‘‘‘‘‘‘‘"‘‘MH&!#0‘‘0DE--C!S:6U?>C(N;6%T4$L!‘A0‘
M%‘‘‘‘‘@‘#4F4*=.2IS$,‘@‘‘R‘<‘‘‘T‘‘‘‘‘‘‘‘‘‘‘‘@‘+:!?0\‘‘$))338P
M=&9?<RYM87102P$"%‘‘4‘‘‘‘"‘#X2I0I;-2:G1L"‘‘#@!P‘‘#0‘‘‘‘‘‘‘‘‘‘
M‘"‘‘MH&T$0‘‘0DE--C!T9E]Z+FUA=%!+‘0(4‘!0‘‘‘‘(‘)MZE"F#?’‘#@‘$‘
M‘!‘’‘‘‘)‘‘‘‘‘‘‘‘‘‘‘‘(‘"V@?H3‘‘!(5D%?<RYM87102P$"%‘‘4‘‘‘‘"‘#$

```
M>I0I,3‘.U)4!‘‘‘@!P‘‘"0‘‘‘‘‘‘‘‘‘‘‘"‘‘MH&A%0‘‘2%9!7WHN;6%T4$L!
M‘A0‘%‘‘‘‘‘@‘‘$R4*;NEX+Z%‘‘‘‘Z‘‘‘‘‘H‘‘‘‘‘‘‘‘‘‘0‘@‘+:!71<‘‘’-I
>9VYA;"YM87102P4&‘‘‘‘‘‘T‘#0‘0‘P‘‘"A@‘‘‘‘‘
‘
end
```