Dynamical Model for a Simplified Air Management System

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1 Modeling of heat exchanger

Let the inputs be a hot flow with temperature T_e, pressure P_v, mass flow W_h and a cold flow with temperature T_a and mass flow W_a, and the output hot flow be T_h, pressure P_c, mass flow W_h. Assume a metal with temperature T_x dividing the two flows, with flux Q_h and Q_a entering the metal from the cold and hot sides, respectively. Newton's cooling law gives

$$\begin{aligned} Q_h &= h_h A_h (T_e - T_x) \\ Q_a &= h_a A_a (T_x - T_a), \end{aligned}$$

where the h are heat transfer coefficients and A areas. With C_{air} as the specific heat capacity of air, this results in

$$\begin{aligned} \dot{T}_x &= \frac{1}{M_x C_{metal}} \left(h_h A_h (T_e - T_x) - h_a A_a (T_x - T_a) \right) \\ W_h T_e - W_h T_h &= \frac{h_h A_h}{C_{air}} (T_e - T_x), \end{aligned}$$

where M_x is the mass of the heat exchanger and C_{metal} is the specific capacity of the metal in the heat exchanger.

Note that heat transfer coefficients are functions of the corresponding flow rate.

2 Dynamical model

Next, the dynamical and algebraic equations governing the behavior of the simple air management system test case shown in Figure 1 are summarized. The symbols used in this draft are given in Table 1.

We have the following equations for the simplified air management system:

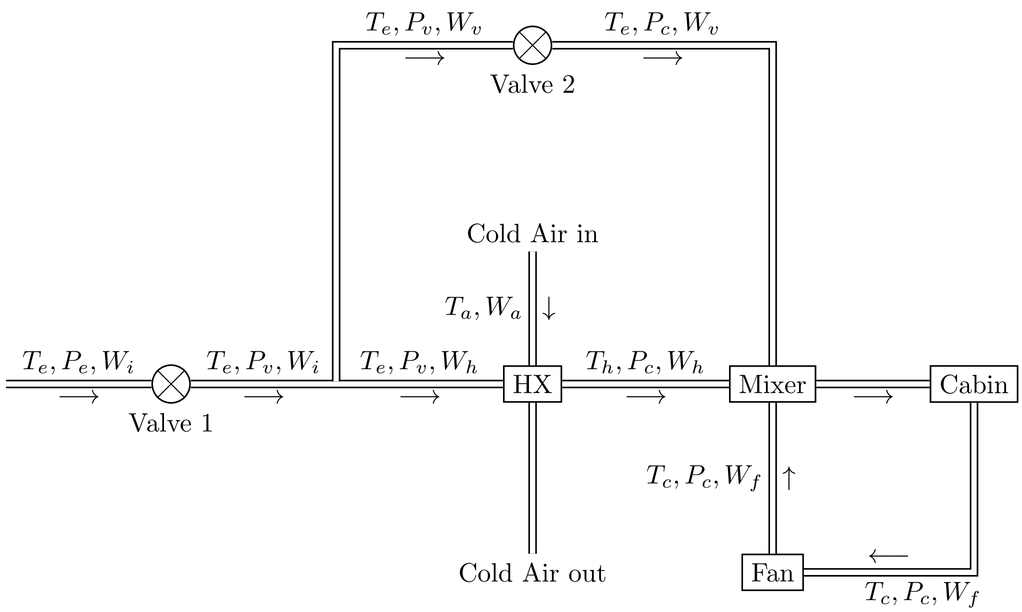

Figure 1: Flow Chart of AMS

1. Mass flow rate equation for valve 1:

$$W_i = \begin{cases} 4.72 \times 10^{-4} \times C_1 \left(P_e + 2P_v\right) \sqrt{\frac{1}{T_e}\left(1 - \frac{P_v}{P_e}\right)} & P_v > 0.5P_e \\ 6.67 \times 10^{-4} \times C_1 P_e \sqrt{\frac{1}{T_e}} & P_v \leq 0.5P_e \end{cases}. \quad (1)$$

2. Mass flow rate equation for valve 2:

$$W_v = \begin{cases} 4.72 \times 10^{-4} \times C_2 \left(P_v + 2P_c\right) \sqrt{\frac{1}{T_e}\left(1 - \frac{P_c}{P_v}\right)} & P_c > 0.5P_v \\ 6.67 \times 10^{-4} \times C_2 P_v \sqrt{\frac{1}{T_e}} & P_c \leq 0.5P_v \end{cases}. \quad (2)$$

3. Equations for the fork (i.e., the point where the pipe splits into two):

$$\begin{aligned} P_v - P_c &= \frac{K}{2\rho_{air} A_{HX}^2} W_h^2 \\ \dot{p}_v &= \frac{RT_e}{MV_{fork}}(W_i - W_v - W_h), \end{aligned}$$

   where A_{HX} is the cross-sectional area of the heat exchanger, K a constant and ρ_{air} the density of air, which is given by

$$\rho_{air} = \frac{M(P_v + P_c)}{R(T_e + T_h)}.$$

   M is the molar mass of air and V_{fork} the volume of the air in the fork.

4. Equations for the heat exchanger:

$$\begin{aligned} \dot{T}_x &= \frac{1}{M_x C_{metal}} \left(h_h A_h (T_e - T_x) - h_a A_a (T_x - T_a)\right) \\ W_h T_e - W_h T_h &= \frac{h_h A_h}{C_{air}} (T_e - T_x). \end{aligned}$$

5. Equation for the cabin:

$$\frac{MP_eV_c}{R}\frac{\dot{T}_c}{T_c} = (T_e - T_c)W_v + (T_h - T_c)W_h + \frac{Q_{passenger}}{C_{air}} + \frac{\Delta Q}{C_{air}}, \quad (3)$$

   with $Q_{passenger}$ as the heat flux from passengers, ΔQ the heat flux from sunlight et.c. in the cabin.

3 Limitations of the model

The model does not currently account for valve dynamics, hysteresis in the valves, static non-linearities for the valves. Uncertainties are lumped into a single term in the equation for the cabin temperature.

References

[1] Tu et al, "Dynamic Simulation of Aircraft Environmental Control System Based on Flowmaster", JOURNAL OF AIRCRAFT, Vol. 48, No. 6, NovemberDecember 2011.

[2] Incropera et al, "Fundamentals of heat and mass transfer", John Wiley & Sons, pp. 676-680.

[3] Boeing: 777-200/-200ER Technical Characteristics, http://www.boeing.com/boeing/commercial/777family/pf/pf_200product.page

[4] Shang, et al., "Development of High Performance Aircraft Bleed Air Temperature Control System With Reduced Ram Air Usage", IEEE transactions on control systems technology, Vol. 18, No. 2, march 2010.

Symbol	Unit	Description
States		
P_v	kPa	Outlet air pressure of valve 1
T_x	K	Temperature of metal in heat exchanger
T_c	K	Temperature of cabin
Controllable variables		
C_1		Valve coefficient for valve 1
C_2		Valve coefficient for valve 2
W_a	kg/s	Mass flow rate of cold inflow in HX
Observable variables		
P_e	kPa	Pressure of the air from the engine
T_a	K	Temperature of cold inflow in HX (ambient air)
Variables (function of altitude or flight mode)		
T_e	K	Temperature of the air from the engine
P_c	kPa	Pressure of the cabin
W_f	kg/s	Mass flow rate passing through the fan
T_a	K	Temperature of the ambient air
$Q_{passenger}$	W	Heat flux generated by passengers in the cabin
ΔQ	W	Heat flux transferred from the environment to the cabin
Other derived variables		
W_i	kg/s	Incoming mass flow rate of the air from the engine
W_v	kg/s	Mass flow rate of the air that goes through valve 2
W_h	kg/s	Mass flow rate of the air that goes through the HX
T_h	K	Outlet air temperature of the HX
$h_h(W_h)$	$\mathrm{W/m^2K}$	Heat transfer coefficient of hot side of HX
$h_a(W_a)$	$\mathrm{W/m^2K}$	Heat transfer coefficient of cold side of HX
Constant		
M	g/mol	Molar mass of air (28.97)
R	$\mathrm{J/(mol \cdot K)}$	Ideal gas constant(8.31)
C_{air}	J/kgK	Specific heat capacity of air
C_{metal}	J/kgK	Specific heat capacity of metal in HX
M_x	kg	Mass of metal in the heat exchanger
V_{fork}	$\mathrm{m^3}$	volume of the fork
V_c	$\mathrm{m^3}$	Volume of the cabin
A_{HX}	$\mathrm{m^2}$	Cross-sectional area of HX
A_h	$\mathrm{m^2}$	Surface area of air/metal interface on hot side of HX
A_a	$\mathrm{m^2}$	Surface area of air/metal interface on cold side of HX

Table 1: The symbols used in the problem formulation